A Study of Functional Testing With SFF Parts

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Abstract

A study of functional testing with SFF parts is presented. The study includes the use of traditional similitude methods as well as advanced similitude methods for predicting product performance through prototype testing. Cantilever beams created from the selective laser sintering process are used to predict the static deflection of both aluminum and polycarbonate beams. The results of 20 experiments using various geometric, loading, and material configurations are presented. Results of the study suggest that a coupling between geometry and material properties exists in SLS parts. Possible sources for the coupling are discussed. An approach for establishing the functional relationship between material and geometry is outlined.

I. Introduction

Advances in virtual prototyping techniques, including numerical modeling and simulation tools, have increased the engineer's ability to quickly analyze a product's performance early in the design cycle. Improved simulation tools have been key in reducing cycle times and improving product quality. Virtual prototypes can often be used in place of physical prototypes in certain stages of the design process, thus saving tremendous time and cost during product development.

Despite advances in virtual prototyping techniques, however, the range of physical phenomena that can be accurately modeled and simulated numerically is still limited. For example, the steady-state temperature of a thermal system, composed of heat sinks and cooling fans, is often difficult to predict using virtual methods only (Nadworny, 1995; Fisher, 1997). Such limitations in virtual modeling require that physical models be used to refine and validate numerical models. Physical models are also important in capturing unique physical behaviors or anomalies that are not accounted for in virtual models. Virtual prototyping techniques, while maturing rapidly, have still not advanced to the point of replacing physical prototypes entirely. In this light, the research presented in this paper is centered on *experimental* methods that are intended to be used as a compliment to, rather than as a substitute for, virtual prototyping techniques.

A serious drawback to functional testing with physical prototypes is the extensive time and cost involved in building and testing the physical part. The fabrication effort for physical parts can be reduced dramatically through solid freeform fabrication (SFF) techniques. While the "fit" and "form" of a product can be accurately evaluated with SFF parts, the amount of functional information that can be obtained is still limited. The limited functional information available from SFF parts is due primarily to the limited number of materials that can be used in SFF processes.

Figure 1 shows a conceptual representation of the current capabilities of SFF in evaluating products along three primary axes: Fit, Form, and Function. The goal of the research presented in this paper is to increase the functional information that can be obtained from SFF parts. By increasing the functional information available from SFF parts, physical prototypes can be constructed with SFF techniques and tested in a fraction of the time that is required for traditional approaches.

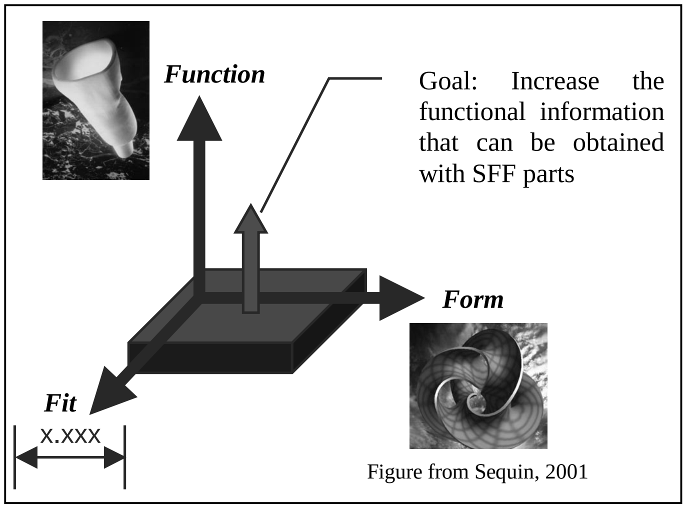


Figure 1. Current capabilities of SFF in evaluating products

Products that contain complex geometry are especially suited for this approach to functional testing because of the extensive fabrication effort that is required from traditional prototyping techniques, such as NC machining. The advantage of SFF techniques over traditional methods increases as part geometry becomes more complex, as illustrated in Figure 2.

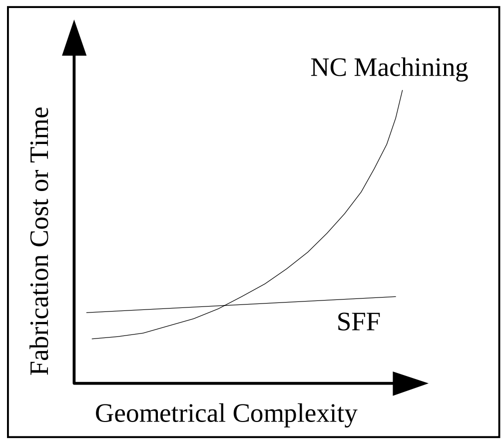


Figure 2. Geometric Complexity vs. Cost

Two main approaches exist for obtaining better functional information from SFF parts:

1. Improve the base materials and/or processes in order to obtain the same properties as those of the part.
2. Correlate properties of existing materials to those of the part in order to predict product behavior.

Significant research has been dedicated to both of the approaches mentioned above. The first approach deals with improving and expanding SFF systems, while the second approach seeks to utilize what is already in place. The second approach is more flexible in its ability to predict behavior for a wide range of parts with different materials, and is the approach taken for the research presented in this paper.

II. Background

Correlating the behavior of two systems can be accomplished by using either dimensional information from the systems or by using empirical data, as described in the following sections.

2.1 Traditional Similarity Method, TSM

The traditional similarity method (TSM), also known as dimensional analysis, is based on the Buckingham Pi theorem. The Pi theorem (Bridgman, 1931) states that a complete equation written in terms of dimensional system parameters d_j, $j = 1, \ldots, n$, can be recast in terms of dimensionless parameters π_i, $i = 1, \ldots, N$, where $N < n$. In equation form,

$$g(d_1, d_2, \ldots, d_n) = 0 \;\;\Rightarrow\;\; f(\pi_1, \pi_2, \ldots, \pi_N) = 0 \tag{1}$$

Two systems that are governed by the same set of dimensional parameters can be correlated by considering corresponding sets of dimensionless parameters. For two systems (a product, *p*, and a model, *m*) the dimensionless parameters can be represented as

$$\begin{aligned} f(\pi_{p,1}, \pi_{p,2}, \ldots, \pi_{p,N}) = 0 \\ f(\pi_{m,1}, \pi_{m,2}, \ldots, \pi_{m,N}) = 0 \end{aligned} \tag{2}$$

or, in terms of a particular parameters of interest, say *X*, as

$$\begin{aligned} \pi_{p,X} &= f(\pi_{p,1}, \pi_{p,2}, \ldots, \pi_{p,N-1}) \\ \pi_{m,X} &= f(\pi_{m,1}, \pi_{m,2}, \ldots, \pi_{m,N-1}) \end{aligned} \tag{3}$$

For these two corresponding systems, the TSM states that $\pi_{p,X} = \pi_{m,X}$ if $\pi_{p,i} = \pi_{m,i}$ for all $i = 1, 2, \ldots, N-1$. (Many references exist which show systematic derivations of dimensionless parameters from sets of dimensional parameters. See for example Barr, 1979 or Langhaar, 1951.)

As a simple example, consider the cantilever beam shown in Figure 3. Suppose we are interested in the deflection of the beam under a static load. A relationship among dimensional parameters can be written as

$$\delta = g(E, t, w, L, F) \tag{4}$$

where δ = static beam deflection
E = Young's modulus
F = applied load
t, w, L = beam dimensions, as shown in Figure 3

Equation (4) can be recast in dimensionless form as

$$\frac{\delta}{t} = f\left(\frac{w}{t}, \frac{L}{t}, \frac{F}{t^2 E}\right) \tag{5}$$

Note that this set of dimensionless parameters is not necessarily unique, but is just one of several possible sets.

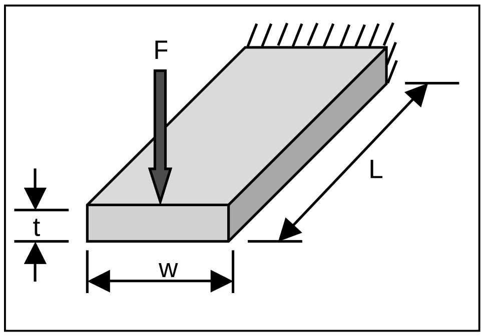


Figure 3. Cantilever Beam

A model beam (*m*) that is similar to the product beam (*p*) (i.e. is governed by the same set of dimensional parameters) can now be used to predict the deflection of the product beam as

$$\frac{\delta_p}{t_p} = \frac{\delta_m}{t_m} \tag{6}$$

if

$$\frac{w_p}{t_p} = \frac{w_m}{t_m}, \quad \frac{L_p}{t_p} = \frac{L_m}{t_m}, \quad and \quad \frac{F_p}{t_p^{\,2} E_p} = \frac{F_m}{t_m^{\,2} E_m} \tag{7}$$

Equation (6) is known as the TSM *prediction equation*, and Equation (7) is known as the *similarity constraints*. The model beam must be constructed so that all of the similarity constraints are satisfied in order for the prediction equation to be valid. Note that scale factors derived from the similarity constraints are assumed to be constant over the range of interest. For example, the scale factor for the applied force on the model ($F_m = \frac{t_m^{\,2}}{t_p^{\,2}} \frac{E_m}{E_p} F_p$) assumes constant ratios for beam thickness and for Young's modulus. If these ratios are not constant, the system is said to be *distorted*. A model beam that does not have a constant value for Young's modulus, for example, will cause the system to become distorted. The behavior of a distorted system cannot be predicted accurately with the TSM.

2.2. Empirical Similarity Method, ESM

The empirical similarity method, ESM, provides a means of correlating distorted systems. The fundamental concept of the ESM is shown in Figure 4. Unlike the traditional method, which relies solely on dimensional information to correlate systems, the ESM utilizes a simplified specimen pair to construct an empirical correlation between systems. The model specimen (*ms*) is a geometrically simplified version of the model, while the product specimen (*ps*) is a geometrically simplified version of the product. The ESM uses measured values from the model specimen, the product specimen, and the model to predict the behavior of the product. In other words, $x_p = f(x_m, x_{ms}, x_{ps})$. The ESM assumes that:

1. The model and the model specimen can be tested to determine the state variation caused by pure geometric changes (G).
2. The model specimen and the product specimen can be tested to determine the state variation caused by pure non-geometric (material and loads) changes (M).

The state of the product can be predicted by multiplying the state of the model by M or by multiplying the state of the product specimen by G, as shown in Figure 4. A basic assumption of the ESM is that M and G are independent.

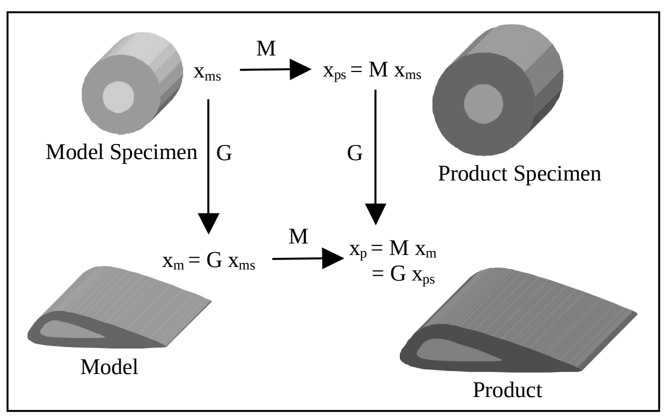


Figure 4. Empirical Similarity Method.

The transformation matrices G and M can be determined with either a circulant matrix approach or with a pseudoinverse approach (since one cannot take the inverse of a vector directly). The pseudoinverse approach for determining M and G can be summarized as follows:

$$
\begin{aligned}
x_{ps} &= Mx_{ms} \quad \Rightarrow \quad M = x_{ps}x_{ms}^{+} \\
x_m &= Gx_{ms} \quad \Rightarrow \quad G = x_m x_{ms}^{+} \\
\textit{where} \quad x_{ms}^{+} &= (x_{ms}^{T}x_{ms})^{-1}x_{ms}^{T}
\end{aligned}
\tag{8}
$$

The advantage of the ESM approach over the TSM is that system distortions due to material properties are captured in the material transformation matrix (M) and system distortions due to geometry are captured in the geometry transformation matrix (G). This procedure is contrasted with the TSM which relies solely on dimensional information to correlate systems, with no means of compensating for system distortion.

The position of the ESM in functional testing of products is between the TSM and Direct Product Tests, as shown in Figure 5. The ESM is presented as a more accurate approach, in general, than the TSM. The ESM is also presented as a better approach for correlating systems with complex geometry whose governing parameters may not be well known, as required by the TSM. Continuing areas of research include identifying the boundaries of the ESM as well as expanding the boundaries of the ESM to replace direct product testing.

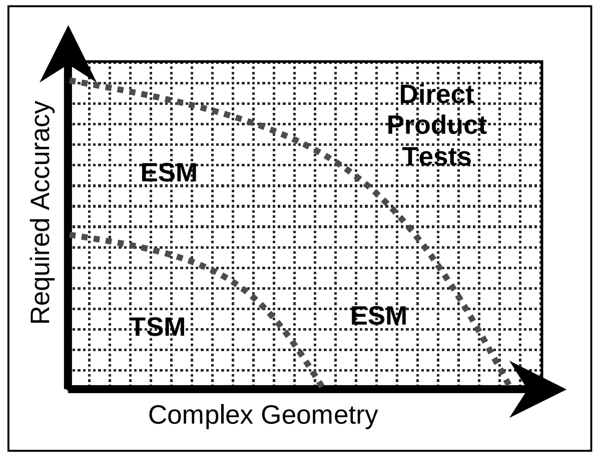


Figure 5. ESM vs. TSM

The ESM approach was validated with a simple cantilever beam example taken from (Cho, 1997). The beams used in the experiment are shown in Figure 6. The product is an aluminum beam with holes. The model is a scaled polycarbonate beam with holes. The model specimen and product specimen are straight beams with no holes, with dimensions and materials as shown in Figure 6. Both the TSM and the ESM were used to predict the deflection of the beam under an applied load. In order to evaluate the different methods, the actual product was also built and tested. (The purpose of both the TSM and the ESM is to predict product performance *without* having to build the product; in this case, the product was built simply to evaluate the accuracy of the predictions). The actual displacement of the beam, along with the ESM and TSM predictions, are shown in Figure 7. The geometric nonlinearities introduced by the holes in the beams cause the TSM to give inaccurate results. The ESM, however, gives a very accurate prediction of the beam deflection.

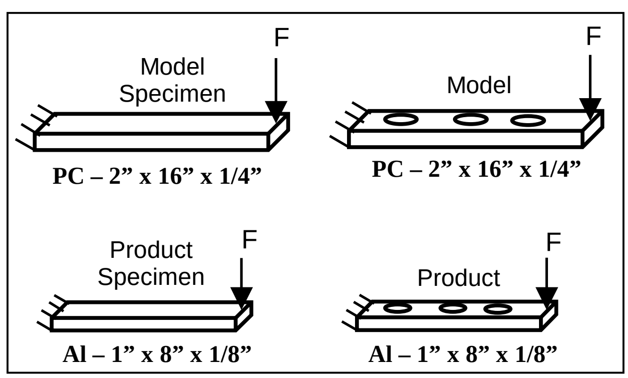


Figure 6. Cantilever Beam Example.

III. Application of ESM to SFF Parts

The feasibility of using the ESM approach with SFF parts was evaluated with a similar cantilever beam experiment. Using SFF techniques to fabricate the model and the model specimen for the ESM introduces additional complexities to the problem, including orthotropic material structures and process-dependent material properties. The initial experiment used a tapered aluminum beam with holes as the product, and a straight aluminum beam without holes as the product specimen (see Figure 8). The model and model specimen were fabricated with DuraForm™, a polyamide-based powder, using the selective laser sintering (SLS) process. The behavior of interest is static deflection under a load applied 10” from the clamped end of the beam. Both the TSM and the ESM were used to predict beam deflection under the load. Again, the actual product was constructed and tested in order to evaluate the accuracy of the predicted results. The results of the test are shown in Figure 9. As shown in Figure 9, the ESM underpredicts the beam deflection by nearly the same amount as the TSM in this case (22% error).

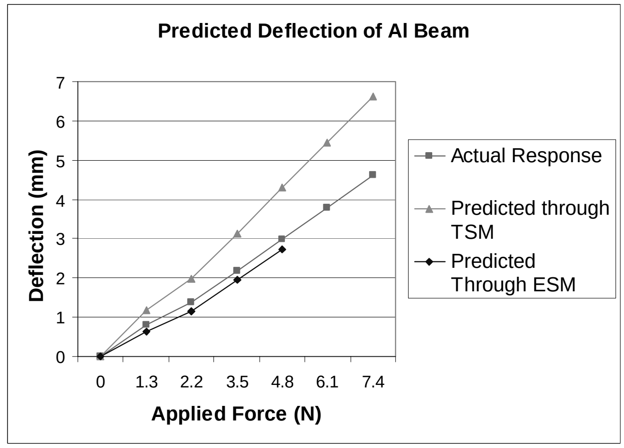


Figure 7. Cantilever Beam Results.

In order to isolate the source of the ESM error, additional experiments were conducted with the following variations:

- Product and specimen geometry were varied
- Product material was varied (aluminum and polycarbonate)
- Loading conditions were varied (load applied at both 4” and 10” from the clamped end of the beam)

A total of 20 experiments were conducted with different combinations of the variables described above. Results of the experiments are shown in Table 1.

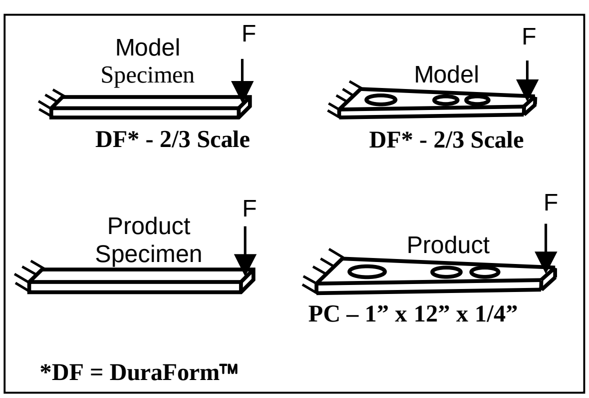


Figure 8. Beam Experiment with SLS Models.

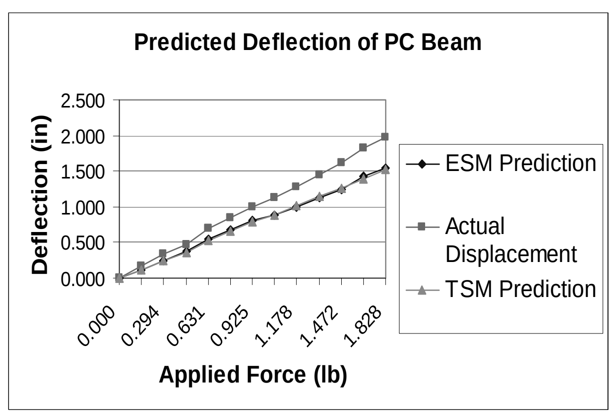
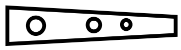
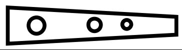
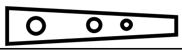
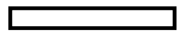


Figure 9. Experimental Results.

Table 1. Experimental Results.

Test	Product/Model Geometry	Specimen Geometry	ESM Prediction Error* (%)			
			PC @ 4”	PC @ 10”	Al @ 4”	Al @ 10”
1			-19	-22	-17	-25
2			+23	+22	+15	+28
3			+23	+22	+32	+21
4			-34	-38	-24	-41
5			-34	-39	-37	-38

*** A negative value indicates a prediction less than the actual value;
a positive value indicates a prediction greater than the actual value.**

A review of the experimental results in Table 1 indicates that the predicted values are very sensitive to changes in geometry, but rather insensitive to changes in product material and application of load. The conclusion that can be drawn from these results is that the material and geometry transformations that are used in the ESM approach are not independent when DuraForm™ SLS models are used. In other words, a coupling appears to exist between material and geometry in SLS parts such that M(G), which violates the fundamental assumption of the ESM.

The SLS process was evaluated for clues as to the source of the coupling between material and geometry. Although many processing parameters could contribute to a coupling between material and geometry, the process evaluation led to the following primary hypothesis:

- Geometric size and complexity affects scan time and, consequently, exposure time between layers of sintered powder to part bed temperature.
- Exposure time of sintered powder to heat may affect the elastic properties (e.g. Young’s modulus) of a sintered beam.
- Elastic properties of a beam directly affect static deflection under an applied load.

The hypothesis is to be tested by setting up a secondary experiment in which the effect of scan time, or exposure time of sintered powder to heat, on elastic properties of sintered parts is evaluated. The results of the secondary experiment will then be used to establish the functional relationship between material and geometry, or M(G). The ESM approach can then be expanded to account for such couplings between material and geometry.

IV. Conclusions

The potential benefits of using SFF parts to obtain functional information on products are tremendous. Replacing direct product tests with tests on SFF models could cut development costs and cycle time significantly. Using SFF techniques to obtain functional information more quickly can also provide more time for product refinement, thus leading to higher product quality.

Similitude methods provide a means of overcoming the current limitations of functional testing with SFF parts. The empirical similitude method has been demonstrated as an effective means of correlating distorted systems that are not compatible with the traditional similarity method. The empirical similitude method, however, operates under the assumption of independence between geometry and material properties. The study presented in this paper suggests that a coupling can exist between material and geometry when SFF parts are used as the model and model specimen.

Future work will include a secondary study aimed at isolating the source of the coupling between material and geometry and establishing the functional relationship M(G). Results from the secondary study can be used to expand the ESM to make it applicable to functional testing with SFF parts.

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