MULTISCALE DESIGN FOR SOLID FREEFORM FABRICATION

Carolyn Conner Seepersad, David Shahan, and Kaarthic Madhavan
Mechanical Engineering Department
The University of Texas at Austin, Austin, TX 78712

Abstract

One of the advantages of solid freeform fabrication is the ability to fabricate complex structures on multiple scales, from the macroscale features of an overall part to the mesoscale topology of its internal architecture and even the microstructure or composition of the constituent material. This manufacturing freedom poses the challenge of designing across these scales, especially when a part with designed mesostructure is part of a larger system with changing requirements that propagate across scales. A set-based multiscale design method is presented for coordinating design across scales and reducing iterative redesign of SFF parts and their mesostructures. The method is applied to design a miniature unmanned aerial vehicle system. The system is decomposed into disciplinary subsystems and constituent parts, including wings with honeycomb mesostructures that are topologically tailored for stiffness and strength and fabricated with selective laser sintering. The application illustrates how the design of freeform parts can be coordinated more efficiently with the design of parent systems.

1. Introduction

The growing field of rapid manufacturing—the use of additive rapid prototyping technologies for fabricating functional parts [1]—presents a host of design challenges. A key challenge is the need for design methods that support exploration of mesostructure (material topology, density, and composition) along with overall product characteristics [2]. An effective "design for rapid manufacturing" methodology is needed to help designers fully exploit the freeform capabilities of RM and create products that cannot be made in any other way, rather than simply creating prototypes of conventionally fabricated parts. Among other capabilities, an effective "design for RM" methodology should offer a means of streamlining design interactions between RM parts with designed mesostructure and parent products or systems [1,2]. It should also help designers avoid excessive redesign of RM mesostructure—often involving expensive topology optimization—in response to changing system- or product-level requirements.

To visualize this challenge, consider the design and RM of a new micro-UAV, as illustrated in Figure 1. The micro-UAV design problem is a multiscale design problem with designers operating on different scales, from the system-level to the subsystem-level (structural, aerodynamics, etc.) to the mesostructural level of topology design for RM. As is typical in industry, a design team decomposes the system-level problem into a set of interconnected and distributed design problems, each solved by an individual or team of designers with very specialized knowledge and tools. The challenge is to manage interdependencies between the teams and negotiate a satisfactory system-level solution. The challenge is formidable because interactions between design teams are often complex, and costly iterations ensue. Iterations are particularly costly in design for RM contexts. At the mesostructural level in this example, the structure of the aircraft is designed for rapid manufacture with selective laser sintering [3]. The wings, for example, are designed with complex, integrated, internal lattice structures that provide lightweight stiffness and strength, along with potential fuel storage capabilities. Such structures are complex to design and require sophisticated topology optimization techniques with time-consuming designer interpretation and translation of results into a manufacturable model of the final design. Repeated iterations of this process are undesirable, but they are common due to the interconnected nature of multiscale design problems. For example, requirements for the mesoscale design problem are supplied by the structural designer, whose problem is coupled with not only the system level problem but also the aerodynamics and fuselage subsystem designs. This high level of coupling leads to repeated iteration between distributed designers who seek to identify satisfactory solutions or respond to changes in system-level requirements.

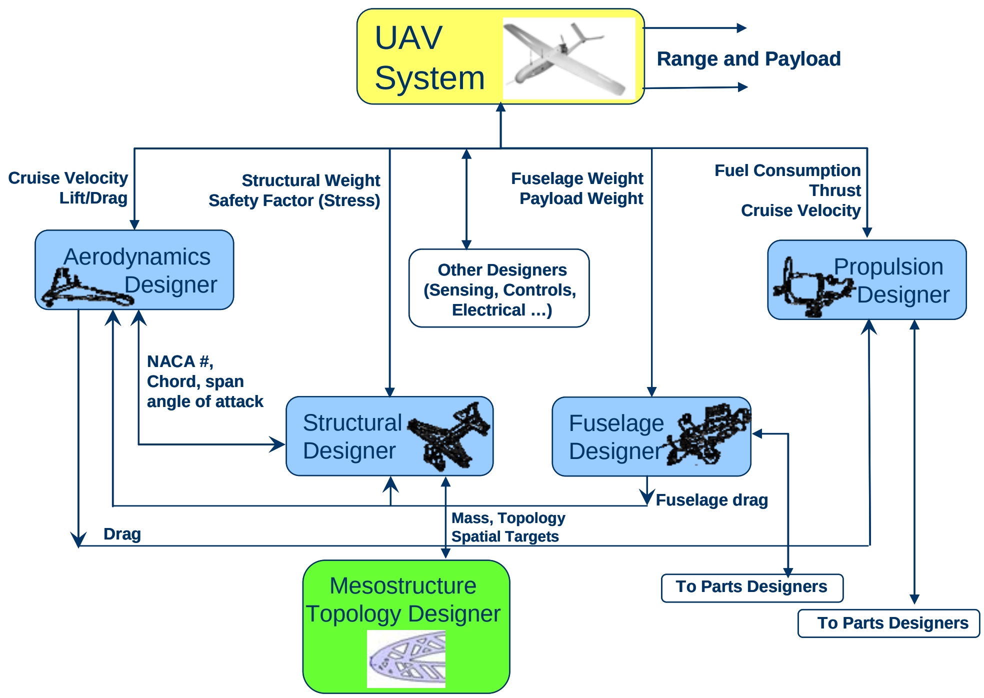

Figure 1. Micro-UAV, Collaborative Design Problem

A methodology is needed: (1) to manage interdependencies between the design of RM parts with customized mesostructures (and possibly microstructures) and the design of parent systems, subsystems, and parts, and (2) to reduce the number of iterative requests for redesign received by mesoscale, rapid manufacturing designers. To address this challenge, we have devised a flexible, set-based method for multiscale design that supports distributed design with minimal iteration between designers. In this paper, we illustrate how this method can be applied to multiscale design for RM problems that incorporate mesoscale topology design. In Section 2, the method is described in the context of the micro-UAV example, and results are presented in Section 3.

2. Method for Multiscale Design for Solid Freeform Fabrication

In recent work, we have proposed a flexibility-based approach to multiscale design [4-6] in which distributed designers manage interdependencies by exchanging targets and Pareto sets of solutions. It is similar to general set-based strategies that have been advocated in the automotive industry [7,8]; in that setting, the exchange of rich sets of solutions (relative to single point solutions) has been shown to increase the diversity of options available for achieving consensus with collaborators and thereby reducing costly iterations. Set-based approaches differ from optimization-based approaches in which point solutions are exchanged in an iterative, automated fashion, under the guidance of one or more optimization algorithms. The difficulty with point-based optimization techniques—such as analytical target cascading [9], simultaneous analysis and design [10], concurrent sub-space optimization [11,12], collaborative optimization [13], and BLISS [14]—is that they require extensive iteration between design teams. In contrast, set-based approaches are intended to reduce iteration between distributed designers by exchanging richer collections of information. In exchange, the solutions tend to be *satisficing* [15] or approximate solutions that are 'good enough' but not necessarily optimal. Set-based coordination strategies have taken several forms, including: (1) robust design techniques for generating ranged sets or intervals of design specifications that can be shared with collaborating designers [16-18]; fuzzy set theory [19,20] for modeling uncertain or imprecise parameters (such as preferences for performance variables) during negotiation; and game theoretic approaches for coordinating the competitive reactions of designers to one another's decisions [21-25]. In the flexibility-based method (FBM) proposed here, designers collaborate by exchanging targets for shared parameters, followed by Pareto sets of solutions that represent achievable tradeoffs between coupled parameters. The sets of solutions provide a diversity of options for achieving system-level performance goals and system-wide feasibility with minimal iteration.

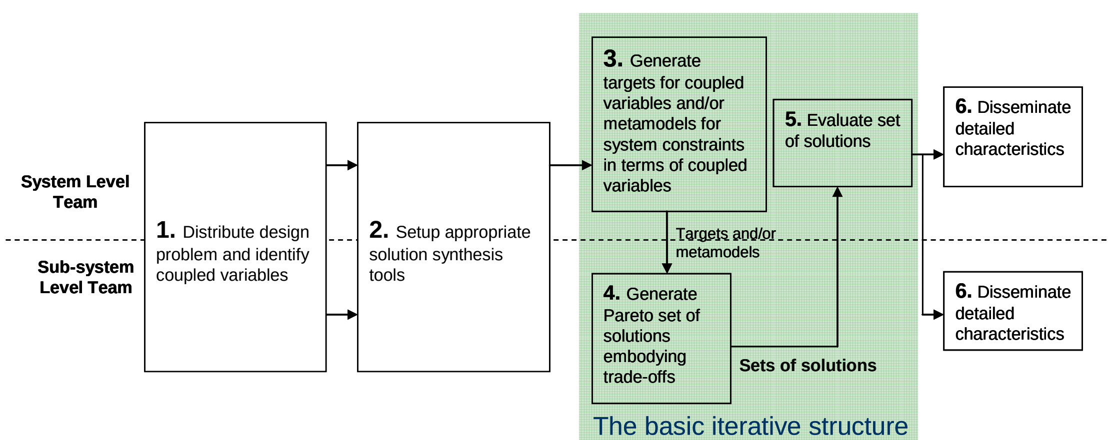

Figure 2. Schematic representation of a flexibility-based approach to multiscale design [5].

2.1 Multiscale Design Exploration

The underlying methodology of the FBM is outlined in Figure 2 and illustrated via application to the micro-UAV problem in Figure 1. As shown in Figure 2, the first step in the FBM is to distribute the problem across scales and disciplines (as shown in Figure 1) and identify the influential shared parameters that couple the distributed design problems. The shared parameters are labeled along the connecting arrows in Figure 1. In Step 2 of the FBM, each designer formulates his/her design problem as a compromise programming problem [26] and constructs appropriate analysis models. Details of the analyses and problem formulations are provided in [6], but a representative design problem is included in Figure 3 for illustration. (Here, the compromise programming problems are formulated as compromise Decision Support Problems, with details of the formulation available in [26].)

Given
Analysis Equations
$Range_{target} = 60$ km; $W_{payload\text{-}target} = 75$ g
Find
Cruise velocity (V), Lift to drag ratio (L/D), Fuel consumption rate ($\dot{m}$), Thrust (T), Wing structural weight ($W_{structural}$), Fuel weight (W_{fuel}), Fuselage weight ($W_{fuselage}$)
Satisfy
Bounds
$28 \le \dot{m} \le 36$ (cc/min) $\quad 0 \le V \le 20$ m/s
$10 \le L/D \le 40$ $\quad 0.01 \le W_{structural} \le 0.03$ kg
$0.5 \le W_{fuel} \le 1.5$ kg $\quad 0.01 \le W_{fuselage} \le 0.3$ kg
$0.5 \le T \le 1.5$ N
Goals

$$\frac{Range}{Range_{target}} + d_1^- - d_1^+ = 1 \qquad \frac{W_{payload}}{W_{payload\text{-}target}} + d_2^- - d_2^+ = 1$$

Minimize

$$Z = f\left(d_1^-, d_2^-\right)$$

Figure 3. Compromise programming problem for system-level design.

In Step 3 of the FBM, the system-level designer solves her compromise programming problem (Figure 3) by finding values of the design variables that satisfy constraints and meet a set of conflicting goals (for maximum range and payload) as closely as possible. To solve this problem, the system-level designer executes a latin hypercube sampling strategy [27] to systematically experiment with values of design variables (lift, drag, thrust, structural weight, etc.) and identify design variable vectors that meet minimum thresholds for range and payload (the system-level designer's primary objectives). Since all of the design variables are also shared parameters that must be coordinated with subsystem-level designers, these satisfactory design variable values become shared parameter targets that are communicated to the subsystem-level designers. System-level targets are illustrated in Table 1 for range and payload thresholds of 60 km and 75 g, respectively.

In Step 4 of the FBM, the subsystem-level designers solve compromise programming problems to generate families of solutions that meet the system-level targets as closely as possible. Sets of cumulative solutions are passed sequentially from the fuselage team to the aerodynamics, structural, and then propulsion teams, in a sequence based roughly on dependencies between them. Detailed descriptions of these intermediate solutions are available in [6]. As a representative sample, the solutions generated by the aerodynamics team are illustrated in the left columns of Table 2. The set of solutions has grown from four to twelve because the subsystem teams generated three Pareto (tradeoff) points for each unachievable target specified by the system-level team. As part of this process, the structural team distributes target requirements to the mesoscale topology designer and receives target-matching sets of solutions in return. The last subsystem team (propulsion) communicates the set of achievable subsystem solutions (in the form of sets of values for the system level shared parameters—lift, drag, thrust, fuselage weight, etc.)—back to the system-level designer.

Table 1. Target values generated by the system-level designer.

Solution #	$W_{structural}$ (kg)	$W_{fuselage}$ (kg)	W_{fuel} (kg)	Fuel consumption rate (cc/min)	Thrust (N)
1	0.1	0.48	1.24	34.9	1.09
2	0.17	0.33	1.15	35.45	1.13
3	0.18	0.40	1.09	28.36	1.07
4	0.19	0.30	1.32	33.68	0.96
Solution #	**L/D ratio**	**Velocity (m/s)**	**$W_{initial}$ (kg)**	**Range (km)**	**$W_{Payload}$ (kg)**
1	26.88	19.79	2	104.11	0.087
2	25	20	2	87.85	0.25
3	22.5	18	2	78.03	0.23
4	26.25	17	2	88.75	0.090

Table 2. Targets from Aerodynamics and Structural Designers, Passed to Mesostructure Topology Designer

	From Aerodynamics							From Structural		
Point	Chord (m)	Span (m)	t (% chord)	Angle of attack (deg)	m (for NACA)	p (for NACA)	Lift (N)	t_{skin} (mm)	n_{ribs}	t_{rib} (mm)
1	0.27	0.55	0.25	2.68	5	7	19.6	2.82	7	2.35
2	0.18	0.86	0.16	5.28	5	7	19.6	2.4	8	1.83
3	0.18	0.97	0.11	6.23	5	7	19.6	0.71	5	1.84
4	0.17	0.80	0.26	5.79	6	6	19.6	1.82	5	4.86
5	0.17	0.83	0.23	5.93	6	6	19.6	1.57	7	1.69
6	0.17	0.83	0.23	5.93	6	6	19.6	1.57	7	1.69
7	0.25	0.64	0.24	3.75	5	7	19.6	1.53	7	2.91
8	0.26	0.81	0.21	0.97	5	7	19.6	0.88	5	1.80
9	0.22	0.83	0.25	2.12	5	7	19.6	2.35	5	4.58
10	0.26	0.56	0.25	6.71	6	6	19.6	2.35	5	4.58
11	0.18	0.97	0.17	6.96	6	6	19.6	0.82	5	1.35
12	0.17	0.84	0.19	5.85	6	6	19.6	0.96	3	1.78

Finally, in Step 5 of the FBM, the system-level designer evaluates the options and selects a solution that maximizes system-level objectives. Final system-level solutions are recorded in Table 3. These solutions typically differ from system-level targets (Table 1) because those targets cannot be achieved exactly by the subsystem designers. Before critically evaluating the final solutions (in Section 3), it is important to describe the mesoscale topology design activities.

Table 3. Final System-Level Solutions.

Solution number	velocity (m/sec)	Thrust (N)	L/D ratio	W_{fuel} (kg)	$W_{fuselage}$ (kg)	$W_{propeller}$ (kg)	$W_{structural}$ (kg)	Range (km)	payload (kg)
1	19.79	1.38	14.27	1.23	0.44	0.1	0.24	73.57	0
2	19.8	0.73	26.88	1.23	0.44	0.1	0.17	83.82	0.05
3	19.92	0.73	26.79	1.23	0.44	0.1	0.15	**84.13**	**0.08**
4	19.98	0.85	23.07	1.15	0.33	0.1	0.21	**72.33**	**0.21**
5	19.99	0.78	25	1.15	0.33	0.1	0.2	**73.77**	**0.22**
6	19.99	0.78	25	1.15	0.33	0.1	0.2	73.78	0.22
7	20	1.24	15.78	1.09	0.26	0.1	0.25	61.89	0.3
8	18.01	0.87	22.5	1.09	0.26	0.1	0.29	**61.33**	**0.27**
9	19.09	0.87	22.63	1.09	0.26	0.1	0.28	**64.17**	**0.28**
10	16.97	1.64	11.99	0.56	0.3	0.1	0.28	21.41	0.77
11	17.02	0.75	26.21	0.56	0.3	0.1	0.2	25.36	0.84
12	20	0.75	26.25	0.56	0.3	0.1	0.19	28.55	0.85

2.2 Mesoscale Topology Design for Rapid Manufacturing

To better understand the activities of the mesoscale topology designer and the impact of the FBM on those activities, it is important to investigate not only the topology design process but also the structural design process with which it is strongly coupled. The structural designer supplies shared parameter targets to the topology designer. The targets are identified by solving the structural design problem; specifically, the structural designer finds values of structural design variables that satisfy a stress constraint and meet the structural weight targets specified by the system-level designer (Table 1). As illustrated in Figure 4, the design variables include the skin thickness (t_{skin}) number of internal support ribs (n_{ribs}) and the thickness of each rib (t_{rib}). The span, chord, wing thickness (t), and NACA aerodynamic profile define the outer geometry of the wing and are inherited from the aerodynamics designer in a set of twelve points (Table 3). Stress constraints are satisfied easily with a nominal skin thickness and an end rib, and the remaining weight is assigned to additional ribs scattered uniformly along the length of the wing. The design process is repeated for each of the twelve target vectors inherited from the aerodynamics designer, and the results are illustrated in the right columns of Table 2.

The focus of the mesoscale topology designer is to design a lightweight truss structure to be rapidly manufactured inside each wing and to replace the simple ribs inserted as placeholders by the structural designer. The topology is designed for maximum stiffness (minimum compliance) subject to the spatial constraints of the wing profile, a maximum mass equivalent to that of the placeholder stiffening ribs, and a load equivalent to five times the weight of the craft to account for steep ascents, descents, and other maneuvers. Accordingly, the structural ribs are replaced with a pair of trusses that extent spanwise inside the wing and attach to the skin with orthogonal cross ribs, as illustrated in Figure 5.

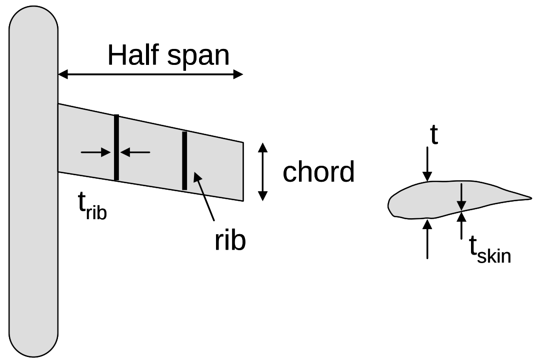

Figure 4. Wing dimensions.

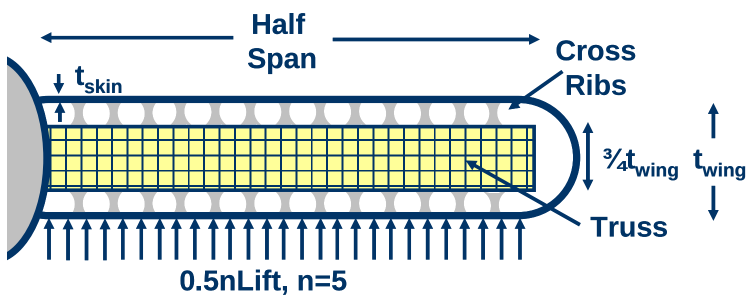

Figure 5. Mesoscale topology design domain.

The topology of each truss is optimized with a two-dimensional SIMP (Solid Isotropic Material with Penalty) topology optimization algorithm [28,29]. The spatial domain (i.e, the cross-hatched rectangle in Figure 5) is discretized into a grid of 2D planar finite elements. The discretization depends on the dimensions of the wing but ranges from 243x8 to 203x20, with each element representing a 2 mm x 2 mm square. The problem is formulated as shown in Figure 6, where ρ_i is the density of element i, E_o is the elastic tensor of a solid element, and p is a constant (typically, p=3) that penalizes the elasticity tensor of intermediate density elements and encourages convergence to solid (ρ=1) and void (ρ=1E-5) areas. A small, nonzero lower bound is assigned to ρ to avoid discontinuities in the FEA formulation. A volume fraction (v_f) constraint is applied to limit the trusses to 50% of the total mass of the ribs, leaving the remaining 50% for support structures to connect the trusses to the skin. Plane strain assumptions are applied with an assumed out-of-plane thickness of 2 mm. A simple filter is applied to avoid checkerboard solutions, and an MMA optimization algorithm [30] is used to solve the problem. Solution requires 2 to 12 hours of processing time on a Pentium 4 PC with 2 GB of RAM, followed by manual translation of the discretized topology into a manufacturable CAD model. Resulting solutions are illustrated in Figure 7.

$$\text{Minimize} \quad Compliance = \sum_{i=1}^{n_{elements}} d_i K_i d_i$$

$$\text{Subject to} \quad v_f \le v_{f-max},\ v_f = \sum \rho_i / n_{elements}$$

$$E_i = \rho^p E^0,\ p = 3$$

$$1E\text{-}5 \le \rho \le 1$$

Figure 6. Topology optimization problem.

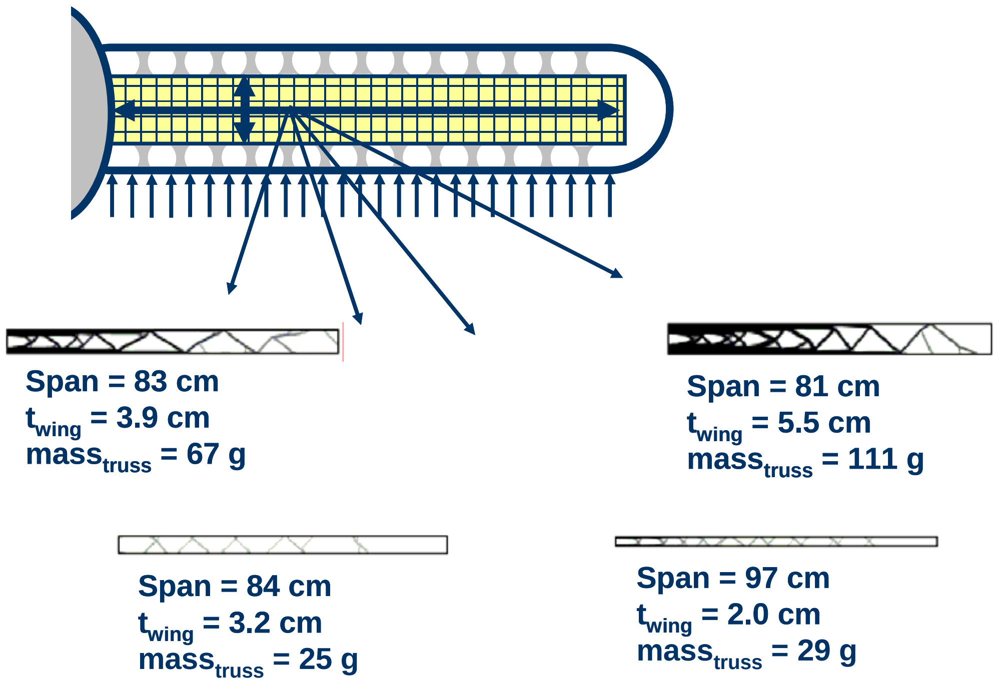

Figure 7. Solutions to topology optimization problem.

Resulting solutions are illustrated in Figure 7. As labeled in the diagram, each solution is aimed at a different target point in Table 2 (with different span, wing thickness, and mass allocations). One interesting feature of the resulting trusses is the enhanced reinforcement near fuselage, where bending stresses are highest. Another interesting feature of the wing architecture is the use of cross ribs to connect the truss structures to the skin. The cross ribs not only enhance stiffness in the transverse direction but also provide flexibility for adapting to small changes in wing dimensions without modifying or re-optimizing the inner truss. In this case, only four separate truss designs were needed since several of the wing geometries were very similar in dimensions (e.g., solutions 4, 5, and 6 in Table 2). This is an example of using flexible product architecture to minimize the impact of changing requirements on a costly design [31].

3. Critical Evaluation of the Multiscale Design Process and Resulting Solutions

From the RM perspective, the primary goal is to reduce the extent of iterative redesign experienced by the mesoscale RM designer, while simultaneously achieving satisfactory system-wide solutions to the overall design problem. Therefore, FBM solutions need to be critically evaluated with respect to their overall solution quality and the frequency of iteration required to obtain them. In Figure 8, targets and final solutions are plotted for the FBM as a function of range and payload, the two system-level objectives. Results are also plotted for a centralized optimization procedure that was implemented for comparison purposes. For the centralized optimization procedure, all of the design problems were combined into a single large optimization problem, and solved in a centralized fashion with a genetic algorithm and a gradient-based, sequential quadratic programming algorithm, in a procedure also known as an All-at-Once (AAO) approach. (To accommodate the large number of iterations associated with this approach, the topology design problem was omitted, and only the system and subsystem level design problems were solved.) In Figure 8, the AAO solutions represent the maximum achievable combinations of range and payload and, therefore, the true Pareto frontier. The target points refer to the target vectors supplied by the system-level designer (Table 1). Satisfactory FBM solutions exhibit range and payload values greater than the satisfactory thresholds of 60 km and 75 g, respectively. As shown in Figure 8, the FBM produces several high-quality solutions within 10% of the maximum achievable system-level performance, as determined by the centralized optimization procedure. Of the twelve solutions evaluated at the system-level at the end of the design process, seven of them met satisfactory thresholds.

From the system-level perspective, the high-quality FBM solutions were obtained with only one global iteration (defined as a cycle between system and subsystem designers) and approximately 90% of the computational expense of the AAO approach. From the mesoscale topology designer's perspective, only one design iteration was required for the FBM, with a set of twelve targets received from the structural designer and consolidated into only four distinct design problems to be solved. In contrast, the AAO approach would have required 1660 iterations, with each iteration corresponding to a separate mesoscale topology optimization problem. This would be an unmanageably expensive design process from the topology designer's perspective.

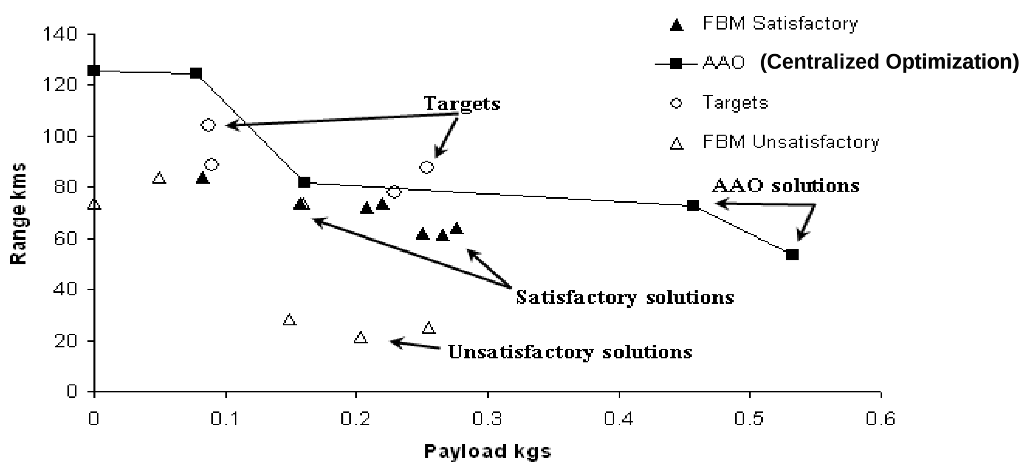

Figure 8. A comparison of FBM solutions with centrally optimized (AAO) solutions.

4. Closure

The set-based FBM is a promising approach for coordinating the design of RM parts with customized mesostructures with the concurrent design of parent systems, subsystems, and parts. For this example, it has been shown to reduce the amount of iteration experienced by mesoscale, rapid manufacturing designers, relative to centralized optimization approaches, without significantly compromising the quality of system-wide solutions. Similar results have been achieved for other problems [4,6].

On-going work involves developing improved strategies for system-wide coordination of sets of solutions. Additional simulation-based and interactive trials are being conducted to refine and validate the approach. Lead time simulations are being developed to predict completion times for minimally iterative set-based approaches and alternative procedures. Preliminary results of discrete event simulations indicate that minimally iterative methods like the FBM reduce not only the expected value of design completion time but also its variability relative to highly iterative, point-based approaches [5]. Finally, from a topology design perspective, robust topology design techniques have proven useful for generating sets of robust, adaptable topologies that are relatively insensitive to processing imperfections and quickly customizable for improved performance (without re-executing the expensive topology optimization process) [32,33]. These techniques can be combined with the FBM to further reduce the need for redesign of mesoscale features for RM.

5. Acknowledgements

We gratefully acknowledge support from National Science Foundation grants CMMI-0632766 and CMMI-0600474, and the Schlumberger Corporation. Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the sponsors.

6. References

1. Hopkinson, N., R. Hague and P. Dickens, Eds., 2006, *Rapid Manufacturing: An Industrial Revolution for the Digital Age*, John Wiley, New York.
2. Dutta, D., F. B. Prinz, D. W. Rosen and L. Weiss, 2001, "Layered Manufacturing: Current Status and Future Trends," *ASME Journal of Computing and Information Science in Engineering*, Vol. 1, pp. 60-71.
3. Beaman, J. J., J. W. Barlow, D. L. Bourell, R. H. Crawford, H. L. Marcus and K. P. McAlea, 1997, *Solid Freeform Fabrication: A New Direction in Manufacturing*, Kluwer Academic Publishers, Boston.
4. Carlos, S., K. Madhavan, G. Gupta, D. A. Keese, U. Maheshwaraa and C. C. Seepersad, 2006, "A Flexibility-Based Approach to Collaboration in Multiscale Design," *11th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Portsmouth, VA. AIAA-2006-7063.
5. Shahan, D., K. Madhavan and C. C. Seepersad, 2007, "The Implications of Alternative Multiscale Design Methods for Design Process Management," *International Conference on Engineering Design*, Paris, France, August 28-31.
6. Madhavan, K., 2007, "A Framework for a Flexibility-Based Approach to Multiscale and Multidisciplinary Design," *M.S. Thesis,* Mechanical Engineering Department, The University of Texas at Austin, Austin, TX.
7. Sobek, D. K., A. Ward and J. K. Liker, 1999, "Toyota's Principles of Set-Based Concurrent Engineering," *Sloan Management Review*, pp. 67-83.
8. Liker, J. K., D. K. Sobek, A. Ward and J. J. Cristiano, 1996, "Involving Suppliers in Product Development in the United States and Japan: Evidence for Set-Based

Concurrent Engineering," *IEEE Transactions on Engineering Management*, Vol. 43, No. 2, pp. 165-178.

9. Kim, H. M., N. F. Michelena, P. Y. Papalambros and T. Jiang, 2003, "Target Cascading in Optimal System Design," *ASME Journal of Mechanical Design*, Vol. 125, pp. 474-480.
10. Haftka, R. T., 1985, "Simultaneous Analysis and Design," *AIAA Journal*, Vol. 23, No. 7, pp. 1099-1103.
11. Sobieszczanski-Sobieski, J., 1988, "Optimization by Decomposition: A Step from Hierarchic to Non-Hierarchic Systems," *Second NASA/Air Force Symposium on Recent Advances in Multidisciplinary Analysis and Optimization*, Hampton, VA. NASA TM-101494. NASA CP-3031.
12. Wujek, B. A., J. E. Renaud, S. M. Batill and J. B. Brockman, 1996, "Concurrent Subspace Optimization Using Design Variable Sharing in a Distributed Computing Environment," *Concurrent Engineering: Research and Applications*, Vol. 4, No. 4, pp. 361-377.
13. Kroo, I., S. Altus, R. Braun, P. Gage and I. Sobieski, 1994, "Multidisciplinary Optimization Methods for Aircraft Preliminary Design," *5th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Panama City, FL. Paper No. AIAA-94-4325. Vol. 1, pp.697-707.
14. Sobieszczanski-Sobieski, J. and S. Kodiyalam, 2001, "BLISS/S: A New Method for Two-Level Structural Optimization," *Structural and Multidisciplinary Optimization*, Vol. 21, No. 1, pp. 1-13.
15. Simon, H. A., 1996, *The Sciences of the Artificial*, MIT Press, Cambridge, Mass.
16. Chen, W. and K. Lewis, 1999, "A Robust Design Approach for Achieving Flexibility in Multidisciplinary Design," *AIAA Journal*, Vol. 37, No. 8, pp. 982-989.
17. Chang, T. S., A. C. Ward, J. Lee and E. H. Jacox, 1994, "Conceptual Robustness in Simultaneous Engineering: An Extension of Taguchi's Parameter Design," *Research in Engineering Design*, Vol. 6, pp. 211-222.
18. Kalsi, M., K. Hacker and K. Lewis, 2001, "A Comprehensive Robust Design Approach for Decision Trade-Offs in Complex Systems Design," *ASME Journal of Mechanical Design*, Vol. 123, No. 1, pp. 1-10.
19. Scott, M. J. and E. K. Antonsson, 1996, "Formalisms for Negotiation in Engineering Design," *ASME DETC/CIE Design Theory and Methodology Conference*. Paper Number: DETC96/DTM1525.
20. Antonsson, E. K. and K. N. Otto, 1995, "Imprecision in Engineering Design," *Journal of Mechanical Design*, Vol. 117, No. 2, pp. 25-32.
21. Lewis, K. and F. Mistree, 1998, "Collaborative, Sequential and Isolated Decisions in Design," *ASME Journal of Mechanical Design*, Vol. 120, No. 4, pp. 643-652.
22. Hernandez, G., C. C. Seepersad and F. Mistree, 2002, "Designing for Maintenance: A Game Theoretic Approach," *Engineering Optimization*, Vol. 34, No. 6, pp. 561-577.
23. Xiao, A., S. Zeng, J. K. Allen, D. W. Rosen and F. Mistree, 2005, "Collaborative Multidisciplinary Decision Making Using Game Theory and Design Capability Indices," *Research in Engineering Design*, Vol. 16, No. 1-2, pp. 57-72.

24. Fernandez, M. G., 2006, "A Framework for Agile Collaboration in Engineering," *PhD Dissertation,* G.W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA.
25. Panchal, J. H., M. G. Fernandez, J. K. Allen, C. J. J. Paredis and F. Mistree, 2005, "An Interval-Based Focalization Method for Decision-Making in Decentralized, Multi-Functional Design," *ASME IDETC/CIE Advances in Design Automation Conference*, Long Beach, CA. Paper Number: DETC2005-85322.
26. Mistree, F., O. F. Hughes and B. A. Bras, 1993, "The Compromise Decision Support Problem and the Adaptive Linear Programming Algorithm," *Structural Optimization: Status and Promise* (M. P. Kamat, Ed.), AIAA, Washington, D.C.**,** pp. 247-286.
27. McKay, M. D., R. J. Beckman and W. J. Conover, 1979, “A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code,” *Quality Engineering*, Vol. 11, No. 3, pp. 417-425.
28. Bendsoe, M. P., 1989, “Optimal Shape Design as a Material Distribution Problem,” *Structural Optimization*, Vol. 1, pp. 193-202.
29. Sigmund, O., 2001, “A 99 Line Topology Optimization Code Written in Matlab,” *Structural and Multidisciplinary Optimization*, Vol. 21, pp. 120-127.
30. Svanberg, K., 1987, “The Method of Moving Asymptotes--A New Method for Structural Optimization,” *International Journal for Numerical Methods in Engineering*, Vol. 24, No. 2, pp. 359-373.
31. Keese, D. A., A. Tilstra, C. C. Seepersad and K. L. Wood, 2007, "Empirically-Derived Principles for Designing Products with Flexibility for Future Evolution," *ASME DETC Design Theory and Methodology Conference*, Las Vegas, Nevada. Paper Number: DETC07-35695.
32. Seepersad, C. C., J. K. Allen, D. L. McDowell and F. Mistree, 2006, “Robust Design of Cellular Materials with Topological and Dimensional Imperfections,” *ASME Journal of Mechanical Design*, Vol. 128, No. 6, pp. 1285-1297.
33. Seepersad, C. C., J. K. Allen, D. L. McDowell and F. Mistree, 2007, “Multifunctional Topology Design of Cellular Materials,” *ASME Journal of Mechanical Design, In Press*.