TOPOLOGY OPTIMIZATION FOR ADDITIVE MANUFACTURING

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Abstract

This paper gives an overview of the issues and opportunities for the application of topology optimization methods for additive manufacturing (AM). The main analysis issues discussed are: how to achieve the maximum geometric resolution to allow the fine features easily manufacturable by AM to be represented in the optimization model; the manufacturing constraints to be considered, and the workflow modifications required to handle the geometric complexity in the post optimization stages. The main manufacturing issues discussed are the potential for realizing intermediate density regions, in the case of the solid isotropic material with penalization (SIMP) approach, the use of small scale lattice structures, the use of multiple material AM processes, and an approach to including support structure requirement as a manufacturing constraint.

Introduction

Topology optimization methods solve a material distribution problem to generate an optimal topology. It is usual for each finite element within the design domain to be defined as a design variable, allowing a variation in density (homogenization, SIMP) [1-4] or void-solid (bi-directional evolutionary structural optimization (BESO)) [5-9]. Other methods exist such as genetic algorithms and level set methods but these are still in their infancy with regards to their suitability to real life problems and so are not discussed here.

Usually, topology optimization methods are used to tackle practical design problems with traditional manufacturing processes in mind, such as casting and machining. Processes where the part is produced by material removal can be described as subtractive processes and processes where the part is produced by a mold can be described as formative processes. These approaches have significant manufacturing constraints that must be taken into account during the design stage to ensure a feasible design. For example, the need for tool access in the case of machining or the need for part removal from a mold in the case of casting or molding. These constraints limit the physical realization of the optimal topology and a compromise has to be made between optimality and ease of manufacture. Typically these constraints are either included in the actual optimization by limiting the topology to feasible designs, or by subsequent simplification of the unconstrained optimization. The former of these is usually preferable, but not all constraints can be included easily in the optimization process.

Additive manufacturing (AM) contrasts to the two aforementioned process classifications in that the part is built up layer-by-layer. AM is a development from rapid prototyping (RP) and aims to produce end-use parts rather than prototypes. To this end, significant efforts have been made in recent years to process metals in addition to polymers, and there are now several commercial metal processes able to produce end-use parts. Like RP, AM usually requires a 3D computer-aided design (CAD) model of the part. This is sliced in a single direction into many very thin slices (cross section profiles). These cross section perimeters are traced either by a laser, electron beam, extrusion nozzle or jetting nozzle and the area contained by the perimeters filled with a hatching pattern. Once a layer has been deposited/melted/cured, the next layer is added. This is repeated until the whole part has been generated.

Due to this layer manufacturing approach, parts of significantly greater complexity can be produced compared with traditional processes and this increased complexity generally does not have a significant effect on the cost of the process. This provides the designer with significantly greater design freedom and enables the built part to be closer to the optimum design than is possible with traditional processes. This paper discusses the application of topology optimization to parts designed for AM, highlighting the main practical difficulties and opportunities for optimization. This work is part of an industrially focused project called Atkins which is investigating carbon reduction through the use of AM and component optimization to reduce weight [10].

Practical Difficulties of Topology Optimization for AM

Mesh Resolution

Topology optimization is a powerful approach for determining the best distribution of material within a defined design domain. Often, the optimized topology is complex and due to manufacturing constraints commonly requires either simplification following the optimization process or constraining of the design space to only allow manufacturable designs. AM enables the manufacture of the topology irrespective of the complexity and the cost of production does not usually increase with complexity. In fact, sometimes the cost can decrease with increased complexity due to reduced support structure requirement. As pointed out in a recent paper by Sigmund [11], optimal stiffness design favors very fine microstructure, which is inherently very complex. Depending on the scale of the designed component, it is difficult to determine the most suitable mesh size in advance to achieve this structure within the manufacturing limits. For traditional manufacturing routes it is usually more expensive to manufacture greater complexity and hence a high degree of complexity is usually undesired. This means that sub-optimal components are manufactured. With AM, there is the capability to manufacture very complex topologies and so there is no reason to prohibit the creation of this complexity.

This leads to some practical difficulties when implementing topology optimization for AM. Firstly, the optimum topology can only be determined if the mesh allows the representation of it. It is well known that as the mesh is refined, further detail emerges and the optimality of the topology improves. For topology optimization, it is usual for each finite element with the design domain to be defined as a design variable, allowing a variation in density (homogenization, SIMP) or void-solid (BESO). Each member of the structure should have at least 2-3 finite elements across its thickness to ensure accurate calculation of the displacement and this has implications for the total number of design variables in the model. Figure 1 shows an example of a topology optimization carried out on an aerospace bracket. Components similar to this have been built using the metal selective laser melting (SLM) process [12] without any requirement for modification. Some support structures are required to support large overhangs, but the topology itself is simply the smoothed optimization result using the SIMP method. Some fine features can be seen in this component, but the minimum feature size for the manufacturing process was far from being utilized. The low minimum feature sizes commonly achievable with AM means that a very high number of design variables are needed to represent the topology of maximum complexity. Currently, this is prohibitive for anything but the optimization of a very small component and so it is no longer the manufacturing stage that is the limiting factor in the realization of optimal designs; it is the design stage.

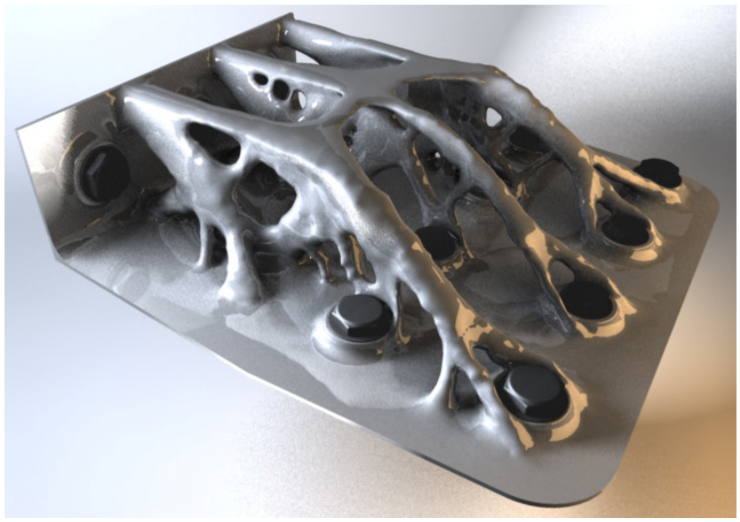

Figure 1: Example topology optimized aerospace bracket for building using a metal AM process.

There are several actions that could be carried out to improve the efficiency of the topology optimization process for AM. Firstly, a hard-kill element elimination approach could be adopted where elements that have remained at very low modulus for a number of optimization iterations are completely removed from the model thereby reducing the number of finite elements. This, though, could encourage a worse result as the elements cannot be returned as the optimization continues. A second approach could be to use iterative remeshing thereby only refining where required and coarsening where a fine mesh is no longer needed. There have been several implementations of this approach in the literature in both 2D and 3D [13-21], and to the author's knowledge a single commercial implementation, in the software TOSCA by FE Design [22]. This commercial implementation is very limited, allowing only refinement and de-refinement in just 2 levels, and does not provide the level of remeshing required for AM optimization. A remeshing method specifically intended for AM has been proposed by [23] which has been coupled with a BESO algorithm. This offers great potential for efficiently taking full advantage of the AM complexity freedom. A third approach could be to use boundary based topology optimization methods such as the level set method [24]. The design variables are then only the boundaries, rather than the finite elements within the volume. Coupling with the XFEM analysis technique, as reported by [25,26], reduces the dependency of the result on the starting mesh.

It could be argued that it would not be worth the added computational expense to improve the optimality of the result only by a modest amount. However, for many practical applications, especially for aerospace, the use phase of the component is by far the most costly in terms of fuel requirement, and even modest weight savings result in a huge overall cost saving over the vehicles' lifespan. This can justify the added computation time at the design stage.

Manufacturing Constraints

While the manufacturing constraints for AM are much less significant than traditional manufacturing routes there are still some that require consideration. Many of the AM constraints could be better termed manufacturing considerations, as they do not necessarily constrain the design. The need for scaffold structures to support large overhangs is dependent on the specific AM process used, as some do not require support structures at all. Up to a point, the processes that require supports, can self-support so long as the overhang is above a particular angle to the horizontal. With some of the metal processes, such as SLM, structures are required primarily to restrict curling/warping of the melted powder due to high temperature gradients, rather than to provide mechanical support. The need for support structures is also dependent on the geometry and often consideration is given to modifying the design to make it self-supporting. The main advantage of this is to reduce the post processing requirement of removing the support structures from the designed component, which is commonly a manual task, but a potential reduction in material usage is also a benefit. Some processes, such as fused deposition modeling (FDM) [27], have water soluble supports which significantly reducing the post processing burden. Other manufacturing constraints are build accuracy, surface finish and z-direction mechanical properties, but these have less relevance to the topology of the component and so will not be discussed here.

As mentioned in the previous section, depending on the specific component application, weight savings can be the primary objective rather than a reduction in manufacturing costs, due to energy use during the component use phase. In these cases, it would not be sensible to increase the weight of the component to reduce manufacturing costs, by reducing the amount of support structure. For applications where the manufacturing costs are more significant, then this could be useful.

As yet, to the authors' knowledge, there has been no research on methods for incorporating specific AM manufacturing constraints into the topology optimization process. The only existing applicable method is the minimum member thickness constraint [28-30] which is applicable to the minimum feature size constraint for the AM processes. This constraint is commonly found in commercial software such as Optistruct by Altair [31] and Nastran by MSC [32]. A maximum overhang constraint would need to be based on the maximum horizontal overhang distance and the angle of the overhang. A maximum thickness constraint as devised by [33,34] and an instance of which has recently been added to Optistruct intended for casting purposes, has some relevance to this issue. By limiting the maximum thickness of the members, it would be expected that this would result in an increase in the quantity of members. This then should reduce the horizontal overhang distance between members, thereby reducing the amount of support structure required. However, it would be difficult to know what specific maximum member thickness value to use in advance and it would likely require several runs to adjust this parameter. It is also unlikely that this would completely eliminate the need for any support material as it does not penalize large unsupported cavities edges.

Recent work by [35,36] has investigated the effect of varying the optimization parameters of a BESO algorithm, specifically the checkerboard filter radius and the evolution rate. This was with the intention of finding the parameters most suitable for AM to increase the complexity of the design and reduce the need for support structures. It was found that the checkerboard filter radius had some effect on the topology complexity, although it did not appear to have enough of an effect to make a significant difference to the requirement for support structures.

For areas of the component that will mate with other components, or that require very high accuracy, post machining may be necessary. Therefore, in these cases a machining constraint would be useful to ensure the tooling can attain access to the relevant features of the component.

First steps towards inclusion of AM specific manufacturing constraints into the topology optimization process are being carried out by the authors. Specifically, this is for the support structure requirement for certain processes, e.g. SLM. There are four main reasons why minimizing the amount of support material required is useful.

1. Support structures require additional material to be used that is usually wasted as it cannot be easily reused by the machine without regrinding it back to a powder.
2. The set up of STL models ready for building requires specification of the build orientation and the subsequent generation and placement of support structures. This commonly requires manual intervention based on the expertise of the technicians.
3. The removal of support structures after building usually requires a significant amount of manual work, especially in the case of metal processes.
4. The requirement for manual removal from the part constrains the geometric freedom of the part as there needs to be hand/tool access.

To include the requirement for the geometry to self support would reduce the need for these aforementioned requirements. The horizontal overhang distance that can self support is dependent on the angle of the edge/face, e.g. hypothetically, for a 30° angle it may be able to self support up to 20mm, but for a 25° angle only up to 15mm. After approximately 45° from the horizontal, the distance that it could self support is not limited. So there are some combinations of angle and horizontal distance that are allowed, but other combinations that are not preferred. Being able to steer the optimization as it progresses to avoid these violations and move towards viable combinations is the objective of this manufacturing constraint.

The BESO algorithm was used for this work because its inherent solid-void representation means that it is easier to identify boundaries than with variable density methods. The implementation of this approach is now explained with an example topology optimization result. At each iteration of the BESO algorithm, an assessment is made of the downward facing edge angles and their horizontal overhang distance. This is done using the following method:

1. BESO topology at iteration x for a simple cantilever plate test case. Build orientation is specified to be in the vertical direction z.

Figure 2: BESO topology optimization result.

2. Identify all cavities.

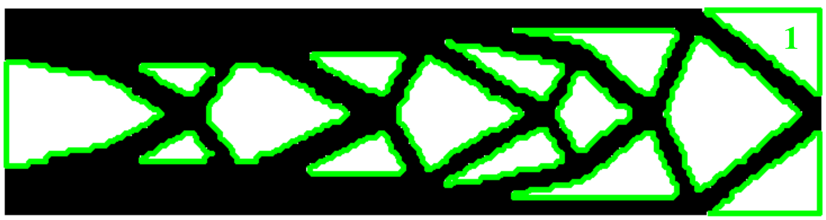

Figure 3: Identification of all cavities in the structure.

3. Filter out cavities that have a width less than a specified value, perpendicular to the build orientation (left to right in this 2D case). This is because even at a horizontal edge angle, the process can still self support a certain distance, so these edges do not need to be considered until they increase in size as would be likely in subsequent iterations as shown in Figure 7.
4. Filter out cavities that do not have any downward facing edges, such as cavity 1 in Figure 3.
5. Identify just the downward facing edges. These are split up by comparing the element coordinates and looking for a negative change in direction.

Figure 4: Identification and splitting of the downward facing edges.

6. Fit a straight line through the data points and calculate the angle from the gradient. This is the approximate angle of the downward facing edge from the horizontal.

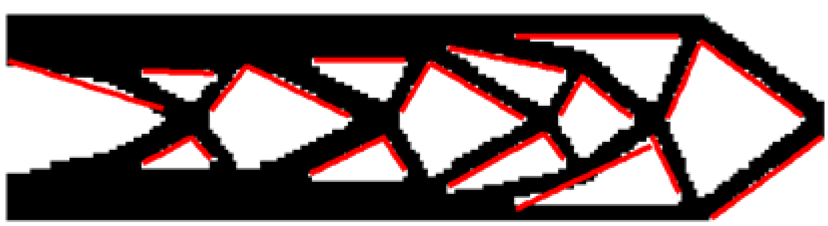

Figure 5: Linear regression fit to the downward facing edge points.

7. Quantify overall violation of self support requirements through use of a penalty function.
8. Combine structural response with penalty function into single objective function.
9. Carry out sensitivity analysis for each design variable on the objective function to aid optimization process.

While practical manufacturing tests are being undertaken on the SLM process to establish what angle-overhang combinations are viable, arbitrary values have been generated that allow the implementation of the method to be evaluated. For each angle, the penalty associated with the horizontal overhang distance is shown in Figure 6. Initially, this function is chosen to be linearly increasing, although this may need modifying depending on performance. The penalty function is therefore defined below the self support threshold as 0, and above the threshold as:

$$\boldsymbol{p = -1 - 0.2a + 0.2h} \quad (1)$$

where a is the edge angle and h is the horizontal overhang distance.

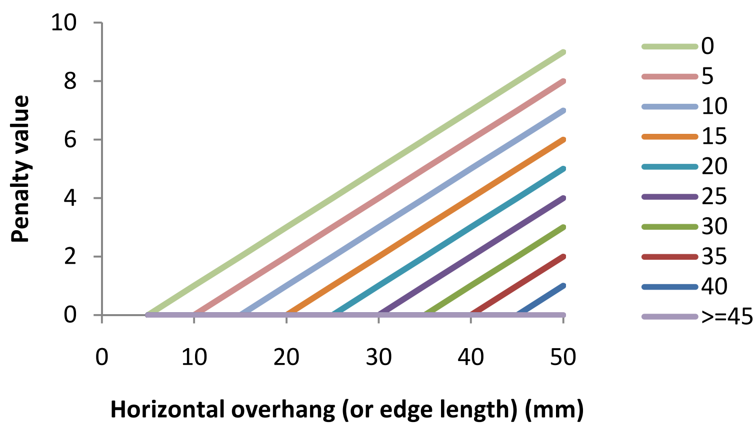

Figure 6 – Initial penalty function for violations of self support requirement.

This manufacturing constraint was not implemented as a direct constraint on the objective function for two reasons. Firstly, there are many possible viable combinations of angle and overhang so it would be difficult to implement this as a constraint; which combination would be the constraint? It would probably be unfeasible to achieve the desired effect using a constraint approach. Secondly, there will probably be instances where it is not necessary for all support structure to be eliminated and so the user should be able to have some control over the strength of the penalty function. By incorporating it into the objective function, a weighting parameter can be included to control this. Currently, this work is in its early stages and the angle measurement algorithm is in the process of being integrated into the sensitivity analysis stage. The analysis of the edge angles does not significantly add to the total computation time required as the bottlenecks are the sensitivity analysis and the FEA. Figure 7 shows iterations of the topology optimization with integrated edge angle measurement.

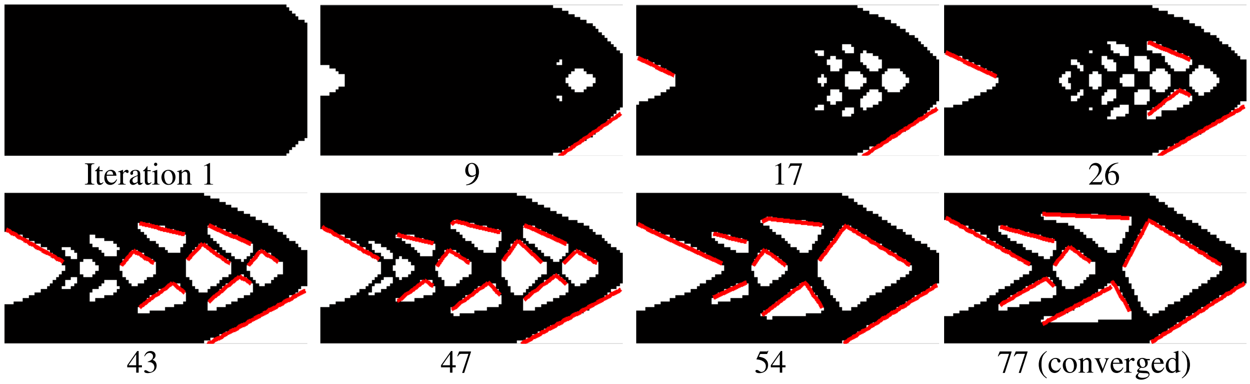

Figure 7 – Iterative downward facing edge analysis during topology optimization, with linear edge approximations plotted in red. Small cavities are ignored due to their inherent self support ability.

Post-optimization Topology Handling

Due to the desired complexity of the resulting topology, there are some practical difficulties to overcome following the optimization stage. These difficulties are commonly encountered when using traditional manufacturing processes which require a simpler topology, but are exacerbated with AM. Following the topology optimization stage, it is usual to smooth the topology to reduce the effects of the element boundaries and to convert the result into a mathematical CAD representation. This stage usually has to be done manually by the designer either by 'tracing' the optimization result or by using some form of feature recognition, which is only practicable for simple topologies. Often, the topology will be simplified at this stage to allow this conversion to be more straightforward or with manufacturing constraints in mind. Due to the high degree of topological complexity when optimizing for AM, manual conversion to CAD is unreasonable, and current automatic methods of conversion have not been designed to handle this level of complexity.

This leads us to question why this conversion stage is really necessary, especially from an AM point of view. Why is a CAD representation of the topology required? For AM in particular, there is little purpose in converting the topology result to CAD, although modifications to the geometry are easier to carry out in CAD software and it makes constructing assemblies with other components more straightforward. A modified workflow for topology optimization for AM is outlined in Figure 8 where the main differences compared with a traditional workflow are in the third stage. The main actions that need to be carried out following the optimization are to interpret/smoothen/modify the optimized topology and to reanalyze the performance with a more accurate FE analysis. It is common to generate a surface mesh from the thresholded isosurfaced topology, commonly a STereoLithography (STL) file. STL files are used as the standard geometry file format for AM and so if further tasks on the optimized topology can be carried out at the STL level it avoids the cumbersome and very difficult conversion to a CAD format. There are several software tools available specifically for handling STL files including Materialise Magics [37], Netfabb Studio [38], and Marcam Autofab [39]. These tools have other functionality, but of use for this task are the smoothening and remeshing functions.

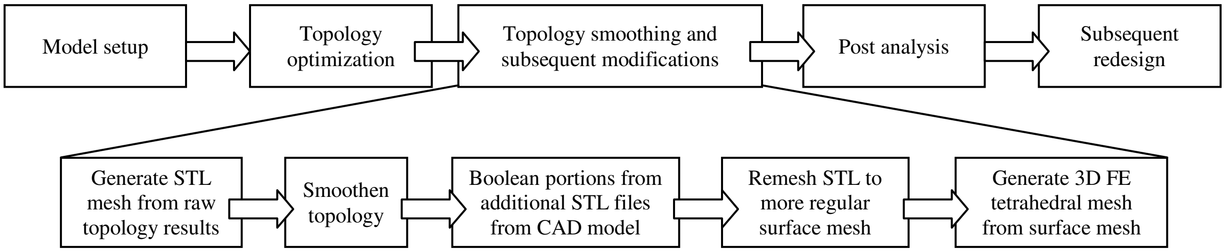

Figure 8: Workflow for topology optimization for AM, with sub-flowchart for the geometry modification stage.

There are commonly built in smoothing functions in topology optimization software such as OSSmooth for Optistruct, but these are only really designed to be used to aid in the interpretation of the optimization result prior to reconstruction in CAD. The STL smoothening tools offer greater flexibility for user control, allowing for local or global smoothening. The geometry can also be modified by either using direct STL manipulation or by generating some portions of the geometry in CAD and then converting to STLs and uniting with, or subtracting from, the existing STL. While this approach is not particularly user friendly, it is a more efficient alternative to converting the topology to CAD and working within that environment.

For reanalysis of the smoothed topology, the remeshing functionality within the STL software is very useful. This allows a mesh with triangles of low uniformity, as shown in Figure 9a, to be converted to a mesh of better quality, as shown in Figure 9b. A solid tetrahedral mesh can be generated from this surface mesh using a standard FE preprocessor, which can be converted to higher order elements if required. The application of loads and boundary conditions is more cumbersome using this approach as there is no associated geometry but a route around this issue is outlined in the flowchart of Figure 10.

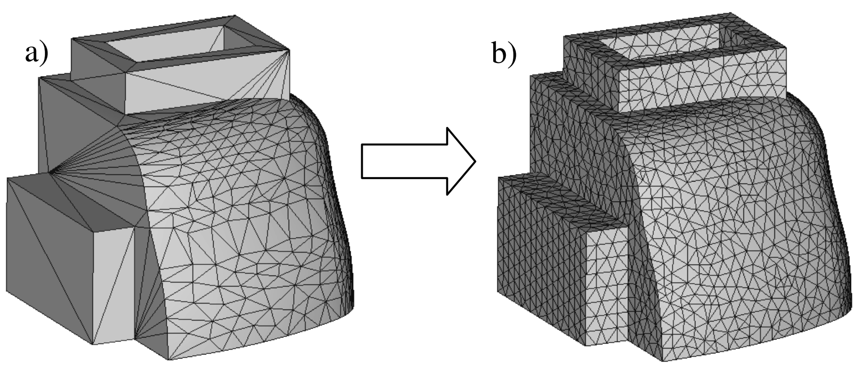

Figure 9: Remeshing an STL to a mesh of better quality for FEA, adapted from [40].

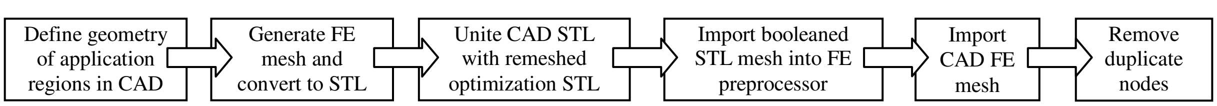

Figure 10: Workflow for retaining geometric associativity for remeshed regions.

Opportunities for Topology Optimization for AM

While the previous section has identified some of the issues with using topology optimization with AM, this section focuses on some of the opportunities AM offers for optimum design. The SIMP algorithm for topology optimization penalizes intermediate densities to encourage discrete void-solid designs. This is because it is assumed that the cost of realizing these intermediate densities is high. However, this artificial penalization means that the optimized topology is less optimal than if the intermediate densities had not been penalized. If there were a way to manufacture these intermediate regions without a direct correlation to cost, then there would be no need to penalize them. It has been shown [41] that microstructures or composites can be used to provide similar mechanical performance to these intermediate density regions. From an AM point of view, this approach of replacing intermediate densities with structures or different materials will now be explored.

Lattice structures

Figure 11 shows a solution to a simple cantilever plate optimization problem using the SIMP method but without the penalization (i.e. SIM) and as would be expected, there are large regions of intermediate density. The first approach to manufacturing these regions is to map the intermediate densities to lattice cells of varying volume fraction, as shown in Figure 12. By interpolating the greyscale result and replacing each pixel/voxel with that from each unit cell, a continuous merging of structure can be achieved as shown in Figure 13.

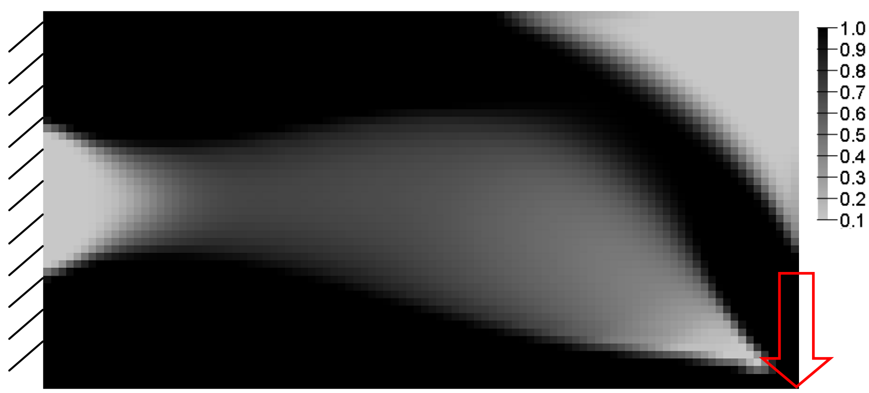

Figure 11: Optimized result for a cantilever plate problem using unpenalized SIMP (i.e. SIM).

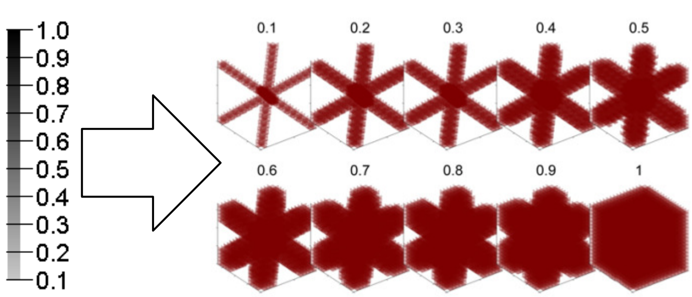

Figure 12: Mapping of variable density to variable volume fraction lattice unit cells.

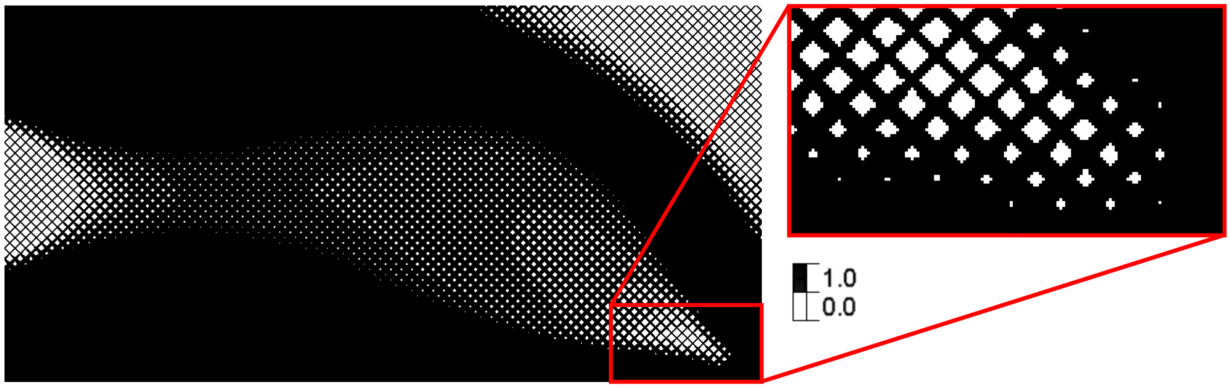

Figure 13: Combined solid and lattice structure by mapping density to unit cells.

Multiple Materials

The intermediate densities from the SIMP method could also be classed as materials of different density. Allowing multiple materials to be used during the design stage can improve the optimality of the resulting topology. There have been several attempts in the literature to tackle topology optimization using multiple materials, either as composites [41], or as discrete regions of different material [42-46]. Hiller and Lipson [46] had AM in mind as a manufacturing route for their multiple material topology optimization results using a multi-material 3D printing technology. Relevant also are investigations into digital materials [47,48] which investigate using stackable voxels of varying designs to construct 3D parts, and methods for designing functional variation of material properties [49,50].

While usually only a single material is used, there are a few AM processes that can handle multiple materials. Commonly used for prototyping, these processes can also be used for end-use parts depending on their application. Extrusion and jetting based processes such as FDM and 3D printing [51] are inherently suited to a multiple material setup. Powder or liquid bed processes such as selective laser sintering/melting (SLS/M) [52,12] or stereolithography (SLA) [53] are less suited to a multiple material setup. Although FDM can currently use two materials, one for support structure and one for part structure, it only uses a single material for the part. Jetting processes use many individual nozzles to jet molten polymer in a similar way to an inkjet printer. Due to the discrete digital nature of individual droplets, it can be envisaged how different materials could be deposited from different nozzles for a single component. A recent process [54] has two 96-nozzle heads each with a different material allowing the deposition of up to 14 blends of the two using droplet combination presets with known mechanical properties. Technically there could be many more than 14 blends, although this would be limited by the size of the part and the resolution of the droplets as they cannot be mixed to create continuous transitions. These materials/blends could be mapped onto a SIMP material interpolation scheme as shown in Figure 14a. Experiments are required to provide a realistic mapping for this.

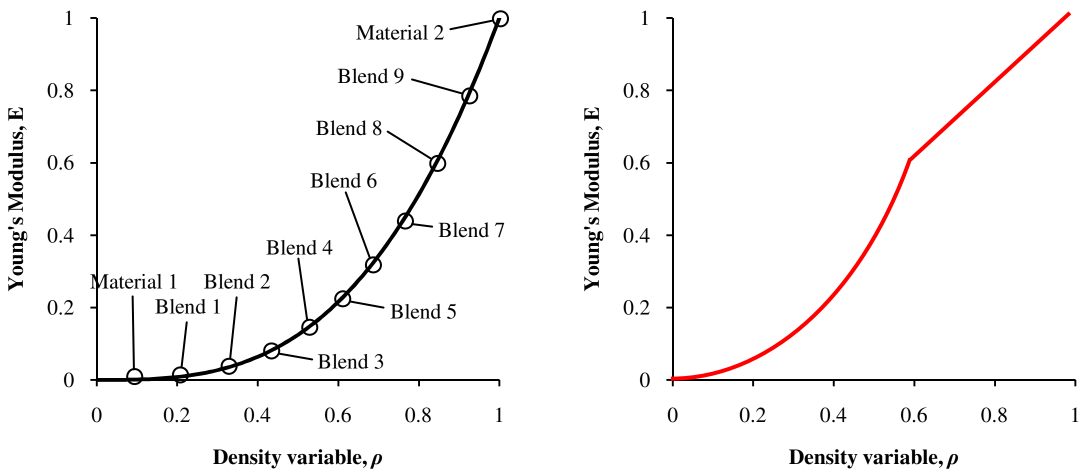

Figure 14: a) SIMP material interpolation scheme with penalization, p = 3, with example corresponding jetted materials/blends, and b) Potential interpolation scheme to include manufacturable functional density variation above 60%.

While not currently a commercially available AM system, a unique approach is variable property printing (VPRP) which can 'dynamically mix and vary the ratios of different materials in order to produce a continuous gradient' [50]. This process uses a 'glue gun' approach using a novel type of nozzle and can use a single material to produce parts with varying material properties.

Processing Parameter Variation

To some extent, the density of the manufactured component can be controlled by varying the processing parameters. In the case of SLM, the laser input power has a significant effect on the porosity of the part. Recently, a two stage approach by Højbjerre [55] has demonstrated precise graded porosity to tailor material properties. This approach was found to be effective for densities above 60% and so while not able to completely control the whole density range, this is useful for a portion of the interpolation scheme. Therefore, the scheme shown in Figure 14b could penalize intermediate densities below 60% but not above.

Conclusions

This paper has summarized the main challenges and opportunities for topology optimization for AM. While it could be currently considered a niche area of manufacturing, its applicability is expected to increase, as it is a relatively new approach to manufacturing and is seeing rapid development. AM offers great potential for physically realizing designs of greater optimality than possible with traditional manufacturing routes. This is enabled by there being no need to penalize complexity due to the layer-by-layer manufacturing approach. This increase in topological complexity does have implications for the design process, namely the large number of design variables required to represent thin members and the difficulties in handling the geometry through the stages of modification, reanalysis and refinement of the design prior to final manufacture. Currently, the only viable way of carrying out the post optimization stages, for a 3D design of high complexity, is to remain in a mesh form throughout the subsequent stages instead of converting to a CAD model. This can be achieved using STL manipulation tools combined with some use of CAD software to assist with certain tasks. The requirement for several AM processes to use support structures for large overhangs provides justification for investigating methods for including this measure into the optimization process to reduce material usage and subsequent post processing.

As well as being able to manufacture components with greater geometric complexity, other opportunities for AM were discussed. These focused on possibilities to realize regions of intermediate density using either small scale lattice structures or by using multiple material processes. Work is needed to investigate this further and correlate the performance of both representations with the variable density isotropic performance.

AM provides a route to physically realize very complex topologies that are of greater optimality than achievable with traditional manufacturing processes. Improvements to the efficiency of the topology optimization methods to allow small and large scale features to coexist without requiring a prohibitive number of design variables are required. The level set approach appears to offer some potential on this issue where the design variables are the boundaries rather than the volume. Tools to aid the designer in handling geometric complexity are also required. It is perhaps unrealistic to expect a panacea of automatic tools to feature recognize and convert complex topology meshes into a mathematical CAD form, but this would be very useful. Until there are further developments in this area, remaining in the mesh form for geometric post-processing appears to be the only realistic way of retaining the level of complexity in the design.

Acknowledgements

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References

[1] Bendsoe M. P., and Kikuchi, N., Generating optimal topologies in structural design using a homogenization method, Computer Methods in Applied Mechanics and Engineering, 71, 1988

[2] Bendsoe M. P., and Sigmund, O., Topological Optimization, Theory, Methods and Application, Springer-Verlag, Berlin, 2004

[3] Rozvany, G. I. N., Zhou, M., and Birker, T., Generalized shape optimization without homogenization, Structural Optimization, 4, 250-252, 1992

[4] Rozvany, G. I. N., A critical review of established methods of structural topology optimization, Structural and Multidisciplinary Optimization, 37, 217-237, 2001

[5] Querin, O. M., Steven, G. P., and Xie, Y. M., Evolutionary structural optimization using a bi-directional algorithm, Engineering Computations, 15 (8), 1031-1048, 1998

[6] Querin, O. M., Steven, G. P., and Xie, Y. M., Evolutionary structural optimization using an additive algorithm, Finite Element in Analysis and Design, 34, 291-308, 2000

[7] Querin, O. M., Young, V., Steven, G. P., and Xie, Y. M., Computational efficiency and validation of bi-directional evolutionary structural optimization, Computer Methods in Applied Mechanical Engineering, 189, 559-573, 2000

[8] Huang, X., and Xie, Y. M., Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method, Finite Elements in Analysis and Design, 43, 1039-1040, 2007

[9] Huang, X., and Xie, Y. M., Evolutionary topology optimization of continuum structures: Methods and Applications, John Wiley and Sons, West Sussex, 2010

[10] atkins-project.com. Project: Atkins – Rapid Manufacturing a Low Carbon Footprint, funded by Technology

Strategy Board (TSB). Accessed 06-05-11.
[11] Sigmund, O., On the usefulness of non-gradient approaches in topology optimization, *Structural and Multidisciplinary Optimization,* 43, 589-596, 2011.
[12] www.mtt-group.com/selective-laser-melting.html. MTT Direct Metals Manufacturing Technology, Accessed 06-05-11.
[13] de Sturler, E., Paulion, G. H., and Wang, S., Topology optimization with adaptive mesh refinement, *Proceedings of the 6th International Conference on Computation of Shell and Spatial Structures IASS-IACM, "Spanning Nano to Mega",* Cornell University, Ithaca, NY, USA, 2008.
[14] James, K. A., Hansen, J. S., Martins, J. R. R. A., Structural topology optimization for multiple load cases using a dynamic aggregation technique, *Engineering Optimization*, 41 (12), 1103-1118, 2009.
[15] Liu, Z., and Korvink, J. G., Adaptive moving mesh level set method for structure topology optimization, *Engineering Optimization*, 40 (6), 529-558, 2008.
[16] Maute, K., and Ramm, E., Adaptive topology optimization, *Structural Optimization*, 10 (2), 100-112, 1995.
[17] Nguyen, T. H., Paulino, G. H., Song, J., and Le, C. H., A computational paradigm for multiresolution topology optimization (MTOP), *Structural and Multidisciplinary Optimization*, 41, 525-539, 2010.
[18] Verani, M., Bruggi, M., Cinquini, C., and Venini, P., Topology optimization and mesh adaptivity, *XVIII GIMC Conference*, Siracusa, 2010.
[19] Wang, S., de Sturler, E., and Paulino, G. H., Dynamic adaptive mesh refinement for topology optimization, *International Journal of Numerical Methods*, 69, 2441-2468, 2008.
[20] Wang, S., *Krylov subspace methods for topology optimization on adaptive meshes*, PhD Thesis, University of Illinois at Urbana-Champaign, 2007.
[21] Stainko, R., An adaptive multilevel approach to the minimal compliance problem in topology optimization, *Communications in Numerical Methods in Engineering*, 22, 109-118, 2006.
[22] www.fe-design.de/en/products/tosca-structure/topology/. FE Design TOSCA Structure.topology software. Accessed 06-05-11.
[23] Aremu, A., Brackett, D., Ashcroft, I., Wildman, R., Hague, R., and Tuck, C., An adaptive meshing based BESO topology optimisation, 9th World Congress on Structural and Multidisciplinary Optimization, Shizuoka, Japan, 2011 (in press).
[24] Wang, M. Y., Wang, X., and Guo, D., A level set method for structural topology optimization, *Computer Methods in Applied Mechanics and Engineering*, 192 (1-2), 227-246, 2003.
[25] Wei, P., and Wang, M. Y., A structural optimization method with XFEM and level set model, *Proceedings of the TMCE*, Izmir, Turkey, 2008.
[26] Wei, P, and Wang, M. Y., Continuum structural optimization with level set model and X-FEM, *8th World Congress on Structural and Multidisciplinary Optimization (WCSMO-8)*, Lisbon, Portugal, 2009.
[27] www.stratasys.com/Solutions/Technology.aspx. Stratasys FDM system. Accessed 06-05-11.
[28] Petersson, J., and Sigmund, O., Numerical Instabilities in topology optimization: a survey of procedures dealing with checkerboards, mesh-dependencies and local minima, Structural Optimization, 16, 1998, 68-75
[29] Poulsen, T. A., A new scheme for imposing minimum length scale in topology optimization, International Journal for Numerical Methods in Engineering, 57, 2003, 741-760
[30] Guest, J. K., Prevost, J. H., and Belytschko, T., Achieving minimum length scale in topology optimization using nodal design variables and projection functions, International Journal for Numerical Methods in Engineering, 61, 2004, 238-254
[31] www.altairhyperworks.com/Product,19,OptiStruct.aspx. Altair Optistruct software. Accessed 06-05-11.
[32] MSC Nastran FE Solver by MSC.Software Corporation, 2 MacArthur Place, Santa Ana, California, http://www.mscsoftware.com, Accessed 11-07-11.
[33] Guest, J. K., and Prevost, J. H., A penalty function for enforcing maximum length scale criterion in topology optimization, Proceedings of the 11th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, 6-8 September 2006, Portsmouth, Virginia
[34] Guest, J. K., Imposing maximum length scale in topology optimization, Structural and Multidisciplinary Optimization, 37, 2009, 463-473
[35] Aremu, A., Ashcroft, I., Hague, R., Wildman, R., and Tuck, C., Suitability of SIMP and BESO topology optimization algorithms for additive manufacture, *21st Annual International Solid Freeform Fabrication Symposium (SFF) – An Additive Manufacturing Conference*, Austin, Texas, 679-692, 2010.
[36] Aremu, A., Ashcroft, I., Hague, R., Wildman, R., Tuck, C., Brackett, D., A parametric analysis of BESO topology optimization algorithm, *Structural and Multidisciplinary Optimization*, (Submitted), 2011.

[37] www.materialise.com/Magics. Materialise Magics software. Accessed 06-05-11.
[38] www.netfabb.com. Netfabb software. Accessed 06-05-11.
[39] www.marcam.de/cms/autofab.82.en.html. Marcam Engineering Autofab software. Accessed 06-05-11.
[40] Brennan-Craddock, J.P.J., *The development of a method to generate conformal structures for additive manufacturing*, PhD Thesis, Loughborough University, 2011 (in press).
[41] Bendsoe, M. P., and Sigmund, O., Material interpolation schemes in topology optimization, *Archive of Applied Mechanics*, 69 (9-10), 635-654, 1999.
[42] Sigmund, O., and Torquato, S., Design of materials with extreme thermal expansion using a three-phase topology optimization method, *Journal of the Mechanics and Physics of Solids*, 45 (6), 1037-1067, 1997.
[43] Yin, L., and Ananthasuresh, G. K., Topology optimization of compliant mechanisms with multiple materials using a peak function material interpolation scheme, *Structural and Multidisciplinary Optimization*, 23, 49-62, 2001.
[44] Wang, M. Y., and Wang, X., “Color” level sets: a multi-phase method for structural topology optimization with multiple materials, *Computational Methods of Applied Mechanical Engineering*, 193, 469-496, 2004.
[45] Huang, X., and Xie, Y. M., Bi-directional evolutionary topology optimization of continuum structures with one or multiple materials, *Computational Mechanics*, 43, 393-401, 2009.
[46] Hiller, J. D., and Lipson, H., Multi material topological optimization of structures and mechanisms, *Proceedings of the 11th Annual Conference on Genetic and Evolutionary Computation (GECCO ’09),* 1521-1528, 2009.
[47] Hiller, J. D., and Lipson, H., Design and analysis of digital materials for physical 3D voxel printing, *Rapid Prototyping Journal,* 15 (2), 137-149, 2009.
[48] Hiller, J. D., and Lipson, H., Tunable digital material properties for 3D voxel printers, *Rapid Prototyping Journal,* 16 (4), 241-247, 2010.
[49] Knoppers, G. E., Dijkstra, J., and van Vliet, W. P., The design of graded material objects, *TNO Science and Industry*, The Netherlands, 2006.
[50] Oxman, N., *Material-based design computation*, PhD Thesis, Massachusetts Institute of Technology, 2010.
[51] www.zcorp.com/en/Products/3D-Printers/spage.aspx. ZCorporation 3D printers. Accessed 06-05-11.
[52] www.3dsystems.com/products/sls/index.asp. 3D Systems SLS process. Accessed 06-05-11.
[53] www.3dsystems.com/products/sla/index.asp. 3D Systems SLA processed. Accessed 06-05-11.
[54] www.objet.com/PRODUCTS/PolyJet_Matrix_Technology/. Objet Polyjet Matrix technology. Accessed 06-05-11.
[55] Højbjerre, K., Additive manufacturing of porous metal components, Proceedings of the 6th International Conference on Additive Manufacturing, Loughborough, July 2011