CHARACTERIZATION AND TESTING OF 3D PRINTED SPROCKETS

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Abstract

Although arguably only 30 years old, 3D printing is already having a tremendous impact upon a broad spectrum of industries, from medical products to consumer goods and nearly every industry in between. The primary driver for growth has been the ability to rapidly prototype components at very low cost. However, as 3D printing technology has matured, industry participants are experimenting with printing components for production and long-term use as opposed to just prototypes.

Chap Research has created a web-based application used to 3D print a “sprocket” – a toothed wheel used in chain-driven systems. This paper details our study to determine the applicability of these plastic 3D printed sprockets in production and for long-term use within certain robotics applications, an area traditionally dominated by metal (aluminum) sprockets.

The study varied a range of design parameters and evaluated whether plastic (PLA) 3D printed sprockets could withstand the torsional stresses and fatigue failure modes present in robotics applications (specifically for FIRST competition robots). This paper also describes two custom test rigs that were built, one to characterize torsional stress and the other for fatigue stress analysis.

Our study indicated that, for our target application, FIRST robotics competitions, 3D printed sprockets were quite sufficient in production and for long-term use. Layer height and infill settings for the 3D prints had the largest impact on the performance of these sprockets. While this is not a study to comprehensively compare performance of 3D printed sprockets to their aluminum counterparts, the big surprise was that 3D printed sprockets performed even better than metal sprockets for fatigue stress modes of failure. Using these results, this study also makes some recommendations on infill and layer height settings for achieving the desired performance for FIRST robotics applications.

The study also showed the relationship between the load applied to a 3D printed sprocket and the number of cycles of continuous operation before a fatigue load related failure occurred and makes a conjecture on a range of load for “infinite life” of operation.

Future extensions of this study may focus on additional fatigue loading tests, expanding to broader sets of load and printer conditions, further optimization of build parameters, varying printing materials, and different 3D printing technologies such as SLS.
A. Background

1. About Sprockets

A sprocket is a toothed wheel that is used with a chain. Sprockets are typically used to drive two parallel axles with one motor and are one of the important components of drive train design. Sprockets vary in size and diameter depending on the target application and the type of chain used. Today most sprockets are made of metal and are mass produced in predetermined shapes and sizes. Customized metal sprockets can also be made using hobbing tools or using a mill.

2. Metal Sprockets

Today’s manufacturing methods reliably produce metal sprockets that have high dimensional accuracy, high strength, and durability and are usable across a wide variety of applications. If one needs a sprocket with any customization or small variants in the design (as is common in design of custom-built robots), the sprockets have to be custom ordered. Such sprockets are typically expensive and also have large lead times for delivery. This leads to slow prototyping speed and typically limits the creativity of those designing systems while increasing expense.

3. 3D Printed Sprockets

3D printing has revolutionized many industries, recently expanding into industrial applications. It provides the ability to prototype quickly and cheaply. 3D printed sprockets look to bring similar benefits to the design and manufacturing of sprockets to robotics and industrial applications. 3D printed sprockets are cheap, easy to create and allow for fast prototyping. Despite these advantages, 3D printed sprockets are relatively new and have not yet seen widespread adoption especially in production or load bearing applications. The primary inhibitors are:

i) Lack of design expertise to create 3D printed sprockets
ii) Concerns about the strength and durability of 3D printed sprockets beyond simple prototypes, especially given they are predominantly made from plastic

4. Key Drivers for Widespread Adoption of 3D Printed Sprockets

Chap Research has developed a program called SprocketR to rapidly and efficiently design sprockets for specific applications. The program generates STL files ready for 3D printing based on user input on the size, number of teeth, chain tension, and sprocket hub type. Due to the general availability of this application and the benefits of 3D printed sprockets, we hope that custom sprockets grow more popular when building robots for FIRST robotics competitions.

However, robot designers need to be confident about the durability and performance of these sprockets in their robots. They will adopt 3D printed sprockets only if these sprockets can perform as well as metal sprockets across a range of loads and not fatigue or incur damage over sustained operation. Future technologies such as carbon fibers can alleviate this concern, but it could be many years before they are full commercialized.

This paper describes our studies to characterize the durability of currently widely available plastic/polymer based 3D printed sprockets for specific robotics applications.
B. Related Work

Our motivation to perform this study was driven by minimal prior art in the arena of characterizing torsional and fatigue stresses in 3D printed sprockets. As per Shigley and Mischke\textsuperscript{6}, Wilfred Lewis introduced an equation in 1892 for estimating the bending stress in sprocket teeth. This equation remains the basis for most metal sprocket design today and is used in the later sections of this study for characterizing the bending stress in 3D printed sprockets. Others have extended the research to deeper work on the analysis of stress and force distributions in metal sprockets. Marshek\textsuperscript{7} presented a spring model analysis for the load distribution in chains and in sprocket teeth and compares the results with those from a traditional force-balance analysis. Naji\textsuperscript{8} provided some findings on analysis of load distribution under torsional load for metal sprockets.

With the greater popularity of 3D printing and their usage in load bearing applications, researchers have begun to make early forays into characterizing them similar to the work done on their metal-based counterparts in the past century. However, studies such as Liu\textsuperscript{9} focus more on the internal structure of the parts and their implications on the strength of these parts. Limited work has been done on specifically characterizing the load bearing capabilities of specialized components such as sprockets.

In an informal survey, researchers have pointed to the need for a custom rig in order to provide empirical characterization of 3D sprocket strength. Such rigs have been used in the past to test moving parts in the field of research medicine. Baums\textsuperscript{10} and Tavakolan\textsuperscript{11} have all described rigs to analyze the load bearing strength of specialized medical devices ultimately targeting limb replacements. To our knowledge, none of the papers describe custom rigs in the field of 3D printing which is an area of significant innovation in this study.

In summary, while there are many research papers broadly in the stress characterization of metal sprockets and gears, and separately on the strength of 3D printed materials, there has been minimal work in the area at intersection of these two areas. There have also been minimal studies in the description of custom rigs that can be employed for the characterization of 3D printed materials in load bearing applications. This study is perhaps among the first in these areas.
C. Operating Parameters

There are a variety of key sprocket design and operating parameters in the real life applications that influenced our study. These parameters include the loads managed, sizes of sprockets needed, and the printer’s manufacturing settings.

1. Loads Managed

The weight of robots in FIRST robotics competitions ranges from 30 lbs. to 120 lbs. In steady state operation, with multiple motors and multiple sprockets, the driving load would be typically divided among many sprockets. However, the torque experienced by a sprocket can be instantaneously quite high when suddenly changing direction, say, going from forward to reverse. Or alternatively, a sprocket might be the key power transfer mechanism to lift a robot off the ground. As a result, these weights are a good proxy for the nature of loads that sprockets could experience in competition. There are three types of load situations depending on the nature of competition or design approach.

   a) Standard FIRST Technology Competition (FTC) robot: 30 lbs.
   b) High-end FTC robot: 50 lbs.
   c) FIRST Robotics Competition (FRC) robot: up to 120 lbs.

This study will evaluate the use of sprockets across this spectrum of loads.

2. Sprocket Design Size Parameters

Robots for FTC fit within an 18” cube, while FRC robots typically fit within a 4’ cube. For these sizes and loads, sprocket dimensions range between 50 and 120 mm in diameter and 16 and 80 teeth. This study will evaluate sprocket strength across these design size values.

3. 3D Printing Alternatives

Today 3D printed parts can be made using FDM\(^{12}\) (Fused Deposition Molding) printers or SLS\(^{13}\) (Selective Laser Sintering) printers. FDM works on an "additive" principle by laying down heated plastic filament material in layers. Selective Laser Sintering (SLS) is an also an additive manufacturing (AM) technique that uses a laser as the power source to sinter powdered material. FDM printers are significantly cheaper than SLS printers, although the latter can produce more complex shapes and structures. Given the lower cost of FDM printers they dominate the market and are the focus of this study.

Within FDM printers, machines could utilize ABS (Acrylonitrile Butadiene Styrene) or PLA\(^{14}\) (Polylactic Acid) as the polymer material. This study only focuses on plastic / polymer parts printed using PLA. A MakerBot Replicator 2X\(^{15}\) is perhaps one of the most popular 3D printers and uses PLA materials and is the printer used in this study. There are obviously parts printed from other materials or technologies that we could study here including ABS or SLS or even futuristic ones such as carbon fibers. Those were not tested and are potential candidates for future studies.

4. Key Sprocket 3D Build Parameters

The two most commonly varied 3D build parameters are:

   a. **Layer Height**: Layer height is one of the key parameters that affects print quality and speed as it sets the thickness of each layer that is being added to generate a print. It is characterized either as the number of layers or the thickness of each layer printed during the additive
manufacturing process. Greater number of layers is the result of a lower layer height. That in turn will result in greater feature resolution and better adhesion to the build plate. This generally delivers better quality and somewhat contributes to higher strength for the part. However, decreasing the layer thickness also means more layers will have to be printed and the time required for 3D printing the part will be significantly increased. One can generally expect lower quality and strength (with all other parameters on the print kept the same) and a faster print with a layer height of over 0.3mm, normal strength and medium time to print with a layer height of 0.2mm and higher strength and slower time to print with layer height of 0.1mm. In this study, we have ranged the layer height between 0.05mm and 0.29mm. Note that during the study we adjusted the number of layers so that each sprocket is the same width, independent upon layer height.

b. **Infill:** Infill is a value represented in percentage from 0% to 100% that shows how filled a sprocket is with material when printed. At 100% infill, there are no gaps within the printed structure providing maximum strength from the material, but also the greatest material cost, slow speed of printing and greatest weight. About 10% percent infill is common for normal uses. In addition to the infill percentage, the infill “pattern” also has an impact upon strength and potentially affects the symmetry of strength of the printed structure. This study varied infill between 10%, 50% and 100% with a common hexagon infill pattern.

5. **Failure Modes Studied**

While there are numerous failure modes, this study will look at two types of failure modes, common for sprockets:

a. **Torsional Failure:** Torsion is the twisting of an object due to an applied torque. It is expressed in newton-meters (N·m) or foot-pounds (ft·lbs.). It acts perpendicular to the axis of rotation and along the motion of rotation of the sprocket. The failure that is caused by excessive rotational stresses from torque on a sprocket is called torsional failure and we will characterize it in ft. lbs.

b. **Fatigue Failure:** Fatigue failure is the tendency of a material to fracture by means of repeated alternating or cyclic stresses of an intensity considerably below the torsional failure point. We will look to characterize the fatigue load bearing capacity of a sprocket by measuring the time of continuous operation before failure.
D. Torsional Test Analysis

For this test we created a set of sprockets and tests with varied parameters in an attempt to determine the effect of those parameters on the torsional strength of the sprocket. Every sprocket created had the same overall thickness – the number of layers was adjusted to accommodate thinner layers. During the test we used #25 chain, so all sprockets were sized to work with that chain.

1. Sprocket Design Parameter Values

Sprocket design and build parameters were tested across the following values:
- Size: Small – 50 mm, Medium – 90 mm, Large – 110 mm
- Infill: 10%, 50%, 100% (Hexagon pattern)
- Layer height: 0.05mm, 0.1mm, 0.15mm, 0.20mm, 0.23mm, 0.26mm, 0.29mm

We were unable to test a set of sprockets representing the complete matrix of the above parameters, so we created a subset of sprockets that we thought best represented these parameters as shown in Table 1.

<table>
<thead>
<tr>
<th>Designator</th>
<th>Size</th>
<th>Infill</th>
<th>Layers</th>
<th>Layer Height (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Small</td>
<td>100%</td>
<td>12</td>
<td>0.24</td>
</tr>
<tr>
<td>S2</td>
<td>Medium</td>
<td>100%</td>
<td>12</td>
<td>0.24</td>
</tr>
<tr>
<td>S3</td>
<td>Large</td>
<td>100%</td>
<td>12</td>
<td>0.24</td>
</tr>
<tr>
<td>S4</td>
<td>Medium</td>
<td>10%</td>
<td>12</td>
<td>0.24</td>
</tr>
<tr>
<td>S5</td>
<td>Medium</td>
<td>50%</td>
<td>12</td>
<td>0.24</td>
</tr>
<tr>
<td>S6</td>
<td>Medium</td>
<td>100%</td>
<td>11</td>
<td>0.26</td>
</tr>
<tr>
<td>S7</td>
<td>Medium</td>
<td>10%</td>
<td>14</td>
<td>0.20</td>
</tr>
<tr>
<td>S8</td>
<td>Medium</td>
<td>100%</td>
<td>13</td>
<td>0.22</td>
</tr>
<tr>
<td>S9</td>
<td>Medium</td>
<td>50%</td>
<td>10</td>
<td>0.29</td>
</tr>
<tr>
<td>S10</td>
<td>Medium</td>
<td>10%</td>
<td>10</td>
<td>0.29</td>
</tr>
<tr>
<td>S11</td>
<td>Medium</td>
<td>100%</td>
<td>14</td>
<td>0.20</td>
</tr>
<tr>
<td>S12</td>
<td>Medium</td>
<td>100%</td>
<td>28</td>
<td>0.10</td>
</tr>
<tr>
<td>S13</td>
<td>Medium</td>
<td>100%</td>
<td>56</td>
<td>0.05</td>
</tr>
<tr>
<td>S14</td>
<td>Medium</td>
<td>100%</td>
<td>21</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 1: Test sprockets.
2. Test Apparatus

We built a test apparatus with the following specifications and layout:

a) Ability to load up to 120 lbs.

b) Load applied by an arm 1 ft. away from the sprocket axle – all our results will be in ft. lbs. The load is measured by the scale attached to the load arm.

c) Provide tension to ensure the chain doesn’t slip while weight is added.

Detailed SolidWorks model, materials list and rig construction instructions are provided in Appendix B.

3. Test Methodology

As Figure 2 above illustrates, the arm is tethered to the ground through a turn-buckle. Rotating the turn-buckle slowly draws the arm down and increases the effective torsional load on the sprocket. This torsional load is then measured using a digital spring scale. During the study, we rotated the turn-buckle really slowly and noted the increasing torsional load measurements on the scale, until the sprocket failed. The scale attached to the turn buckle times arm length (1 ft.) would indicate how much torque was being placed on the sprocket at the time of failure.
4. Test Results

The table below illustrates the torsional load failure torque in ft. lbs. for the sprockets tested and averaged across two independent tests. The failure torque observed in the two tests were between 2% and 8% off each other. The average of their breaking point is below. Separately, we also tested the torsional load failure torque for a comparable sized metal sprocket. We could not create a breakage even past 140 ft. lbs. of torsional load which was the maximum torque load we could create with our test apparatus.

<table>
<thead>
<tr>
<th>Designator</th>
<th>Description</th>
<th>Mean Breaking point (ft. lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Small sprocket with 100% infill with 12 layers</td>
<td>73.27</td>
</tr>
<tr>
<td>S2</td>
<td>Medium sprocket with 100% infill 12 layers</td>
<td>74.73</td>
</tr>
<tr>
<td>S3</td>
<td>Large sprocket with 100% infill 12 layers</td>
<td>76.23</td>
</tr>
<tr>
<td>S4</td>
<td>Medium sprocket with 10% infill 12 layers</td>
<td>27.49</td>
</tr>
<tr>
<td>S5</td>
<td>Medium sprocket with 50% infill 12 layers</td>
<td>63.01</td>
</tr>
<tr>
<td>S6</td>
<td>Medium sprocket with 100% infill 11 layers</td>
<td>71.52</td>
</tr>
<tr>
<td>S7</td>
<td>Medium sprocket with 10% infill 14 layers</td>
<td>29.89</td>
</tr>
<tr>
<td>S8</td>
<td>Medium sprocket with 100% infill 13 layers</td>
<td>78.14</td>
</tr>
<tr>
<td>S9</td>
<td>Medium sprocket with 50% infill 10 layers</td>
<td>45.45</td>
</tr>
<tr>
<td>S10</td>
<td>Medium sprocket with 10% infill 10 layers</td>
<td>24.5</td>
</tr>
<tr>
<td>S11</td>
<td>Medium sprocket with 100% infill 14 layers</td>
<td>81.3</td>
</tr>
<tr>
<td>S12</td>
<td>Medium sprocket with 100% infill 28 layers</td>
<td>94.8</td>
</tr>
<tr>
<td>S13</td>
<td>Medium sprocket with 100% infill 56 layers</td>
<td>N/A</td>
</tr>
<tr>
<td>S14</td>
<td>Medium sprocket with 100% infill 21 layers</td>
<td>90.24</td>
</tr>
</tbody>
</table>

Table 2: Sprocket torsional failure points.

5. Conclusions

a) Impact of Infill:

Infill appeared to have the most significant impact on sprocket torsional stress failure. The chart below plots the torsional failure load for medium-sized sprockets with 12 layers and infill ranging from 10% to 100%.
b) Impact of layer height:

The number of layers positively affects the amount of torque a sprocket could hold although it appears to level off around 20 layers as shown below. This chart plots the torsional failure point for Medium sized sprockets at 100% infill while varying the number of layers (which is the inverse of layer height).

![Graph showing torsional failure load vs. number of layers.]

Figure 5: Torsional failure load vs. infill.

![Graph showing mean torsional failure load vs. number of layers.]

Figure 6: Torsional failure load vs. number of layers.

c) Impact of sprocket size:

The size of the sprocket has only a modest to insignificant impact on the torsional stress load that a sprocket can bear. Torsional failure loads with 100% infill and 12 layers across Small, Medium and Large sprockets are shown in the chart below.
d) **Overall range:**

In general, across all sprocket sizes, median layer height 0.20mm (or 14 layers) and 50% or greater infill, sprockets withstand greater than 50 ft. lbs. of torsional load.

e) **Comparison with metal sprockets**

Given that we could not create a breakage of the metal sprocket even past 140 ft. lbs. of torsional load, we can safely conclude that metal sprockets are materially superior to 3D printed sprockets in terms of torsional strength performance.

6. **Implications and Recommendations for FTC and FRC robots**

a) **FTC robot implications**

i) Tetrix10 axles (typically used in FTC robots) only take about 15 ft. lbs. to bend and therefore bend before the sprockets break. As a result, in all instances where a Tetrix axle is used in FTC robots, the sprockets will withstand more torsional load than the axle. Even if Tetrix axles were not used, given that FTC robots weigh 30 lbs. to 60 lbs. and torsional stress points will be less than 6” from the center of the sprocket, torsional loads will generally be well less than 30 ft. lbs. That would mean most 3D printed sprockets will have adequate torsional load performance.

ii) This study indicates that for FTC applications a 10% infill for sustained sprocket performance will be sufficient.

b) **FRC robot implications**

i) The torque that FRC robots are exposed to will likely be less than 60 ft. lbs. (stress points are generally less than 0.5 ft. away from the center of a robot and according to the rules, the robot MUST weigh less than 120 lbs.).

ii) Infill of 100% and a wide spectrum of layer heights have resulted in sprockets that have withstood torsional loads greater than 60 ft. lbs. and will be more than adequate for FRC applications. Even an infill of 50% should be adequate for most use cases given that such sprockets have a torsional load failure of over 60 ft. lbs.
E. Fatigue Failure Analysis

For this test the goal was to create a set of sprockets and load tests with varied parameters to determine the effect of those parameters on the fatigue strength of the sprocket.

1. Sprocket Design Parameter Values

While there is a vast spectrum of sprockets that can be studied for the effect of fatigue loads, rather than construct a broad experimental design, we decided to start this study with ONE specific sprocket – Large sprocket (diameter – 110mm), 10% infill and 14 layers (0.20mm layer height). The reasons for this choice are as follows:

- Such a sprocket would be at the lower end of torsional load bearing performance as discussed in the earlier section. This sprocket’s performance under fatigue load would then influence the rest of the study on how to potentially expand to a broader set of sprockets.
- Such a sprocket will also be used very commonly for both FTC and FRC purposes.

2. Load Parameter Values

A sprocket in a typical FRC robot will be driven by a DC motor that in steady state draws 5A to a peak of 10A of current from a 12V battery source or in other words encounters a 60W to 120W load. Our plan to analyze fatigue load performance was to assess the time duration of reliable performance of a 3D printed sprocket versus a metal sprocket under such load conditions.

3. Test Apparatus

The test apparatus shown in Figure 8 illustrates the test apparatus built for this purpose. The power supply provides a 12V source that drives the Powered Motor which in turn drives the aluminum sprocket. The metal sprocket then drives a 3D printed sprocket which is connected to a back driven motor. The back driven motor serves as a generator (G) and so the 3D printed sprocket effectively generates current. A bank of resistors connected in parallel serve as a load and draw current from this generator and effectively emulate a robot drive load. We set up a bank of 1Ω, 0.5Ω and 0.33Ω resistors from which we could create resistances ranging from 0.167Ω to 1Ω as shown in Figure 9 and in Table 3 below. The range of resistances was set up to emulate a broader array of load bearing scenarios across which we could then test the relative fatigue stress performance of sprockets.
The resulting resistance ($R_{\text{effective}}$) can be computed as $R_{\text{effective}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$. The resulting resistance achieved (which can then emulate different load scenarios) by turning on / off these three switches is illustrated in Table 3.

<table>
<thead>
<tr>
<th>Switch 1</th>
<th>Switch 2</th>
<th>Switch 3</th>
<th>$R_{\text{effective}}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>On</td>
<td>Off</td>
<td>Off</td>
<td>0.330</td>
</tr>
<tr>
<td>On</td>
<td>On</td>
<td>Off</td>
<td>0.199</td>
</tr>
</tbody>
</table>

Figure 8: Test for fatigue failure analysis.

Figure 9: Resistor bank configuration that draw current from Generator (G).
<table>
<thead>
<tr>
<th>Configuration</th>
<th>On/Off</th>
<th>On/Off</th>
<th>On/Off</th>
<th>Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Off</td>
<td>Off</td>
<td>On</td>
<td>1.000</td>
</tr>
<tr>
<td>4</td>
<td>On</td>
<td>Off</td>
<td>On</td>
<td>0.248</td>
</tr>
<tr>
<td>5</td>
<td>Off</td>
<td>On</td>
<td>Off</td>
<td>0.500</td>
</tr>
<tr>
<td>6</td>
<td>Off</td>
<td>On</td>
<td>On</td>
<td>0.333</td>
</tr>
<tr>
<td>7</td>
<td>On</td>
<td>On</td>
<td>On</td>
<td>0.166</td>
</tr>
</tbody>
</table>

*Table 3: Resistance scenarios from turning on / off the three switches.*

Lower the resistance should result in higher current draw from the generator and hence the greater power load. Configuration 3 (with the highest resistance) should result in the lowest current draw and results in a power generated ($P_{Driven}$) of 32W. We estimate the efficiency ($\epsilon$) of the back driven motor (generator) to be 40% per VEX CIM. From that the load on the sprocket ($P_{Sprocket}$) is calculated as:

$$P_{Sprocket} = \frac{P_{Driven}}{\epsilon}$$

That translates to a load at the sprocket ($P_{Sprocket}$) of 80W which is in the mid-range of the operating range of FRC robots. Other configurations will result in a higher load at the resistors and so will emulate an even greater loads than the one typical for mid-range of FRC motor loads. The next section looks at the broader study of fatigue load performance and the impact on the number of cycles of operation.

*Figure 10: 3D Printed sprocket being tested in above testing rig.*

Detailed SolidWorks model, materials list and rig construction instructions are provided in Appendix C.

4. **Test Methodology**

We set the resistance to Configuration 3 and connected the power source to the drive motor, allowing it to drive the load motor through the test sprocket. We were visually inspecting the 3D printed sprocket (and the metal sprocket) for wear and tear and measuring the time it took for breakage or operational failure.

5. **Test Results**

In the biggest surprise of this study, after **7 hours and 53 minutes**, the metal sprocket failed while the 3D printed sprocket continued to operate. Every one of the metal sprocket’s teeth wore off (Figure 11) while most of the 3D printed sprocket teeth (Figure 12) remained in working condition. This wearing out of the metal sprocket (and the 3D printed sprocket) was quite gradual during the entirety of the fatigue load test.
6. Implications and Recommendations for FTC and FRC Robots

This is a very encouraging result for the use of 3D printed sprockets for FTC and FRC robots. It suggests the following:

- These sprockets will have lesser fatigue load related failures for standard operations even compared to metal sprockets.
- Given that we performed this test with a sprocket that has a low torsional failure point, we can likely conclude that these sprockets are more likely to fail from torsional stress rather than from fatigue failure.
- With regard to manufacturing recommendations, a broad range of infill and layer height will likely be acceptable to meet a desired fatigue failure performance.
F. Maximum Bending Stress Characterization

The modified Lewis equation for characterizing the bending stress ($\sigma$) in a sprocket as provided by ASME Y14.5-2009 is as follows:

$$\sigma = \frac{W^t}{YF}$$

Where:
- $W^t$ = Transmitted load on the sprocket tooth (N)
- $p$ = The circular pitch of the sprocket – 6.52 x $10^{-2}$m (in our experiments)
- $P = \frac{\pi}{p}$ - the diametral pitch of the sprocket
- $F$ = the width of the sprocket tooth at its base (m) – 2.697 x $10^{-3}$m (in our experiments)
- $Y = 0.47$ (as per above ASME standard with teeth count of 60 and $\varnothing = 25^\circ$

We will apply this equation to the estimation of the maximum bending stress in the sprocket for the operation of different numbers of cycles of operation and plot the tradeoff between the maximum bending stress that can be applied on the sprocket and the number of cycles of operation that we can achieve from that. In theory, the tradeoff will look as follows. The dotted line indicates the theoretical maximum bending stress below which the sprocket will have infinite life or cycles of operation.

![Figure 13: Max bending stress vs. Number of cycles of operation of sprocket.](image)

1. Estimation of $W^t$

   The challenge is the estimation of $W^t$ in the Lewis equation which we accomplished using the following steps.

   **Step 1:** Estimate the driven load – $P_{Driven}$. Using a traditional multi meter, we derived the power consumed by the resistors.

   **Step 2:** As described in the previous section, we assumed an overall 40% efficiency ($\epsilon$) of the generator and calculated the load on the sprocket ($P_{Sprocket}$) as:

   $$P_{Sprocket} = \frac{P_{Driven}}{\epsilon}$$
Step 3: Compute the angular velocity ($\omega$) of the sprocket in radians per second. For that, using a non-contact tachometer, determine the RPM at the sprocket ($RPM_{sprocket}$).

$$\omega = RPM_{sprocket} \times \frac{2\pi}{60}$$

Step 4: Compute the torque ($\tau$) at the sprocket tooth

$$\tau = \frac{P_{sprocket}}{\omega}$$

Step 5: Compute the load at the sprocket tooth ($W^t$) at the sprocket tooth

$$W^t = \frac{\tau}{r}$$

For the sprockets used, the radius ($r$) = $5.5 \times 10^{-2}$ m or 5.5 cm.

Step 6: Plug $W^t$ and other sprocket dimensional parameters into the Lewis equation to compute the bending stress and plot against the number of cycles at the time of failure of sprocket.

2. Results

The measured RPM at the sprocket and the measured current, voltage and the power across the different load configurations at the resistor banks and averaged across two different sprockets of identical shape, size and gear ratios is provided below.

<table>
<thead>
<tr>
<th>Configurations</th>
<th>$R_{\text{effective}}$ (Ω)</th>
<th>RPM$_{sprocket}$</th>
<th>Current (A)</th>
<th>Voltage (V)</th>
<th>$P_{\text{Driven}}$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration 1</td>
<td>0.330</td>
<td>3637</td>
<td>12.6</td>
<td>4.4</td>
<td>55.0</td>
</tr>
<tr>
<td>Configuration 2</td>
<td>0.199</td>
<td>3457</td>
<td>10.9</td>
<td>3.7</td>
<td>41.0</td>
</tr>
<tr>
<td>Configuration 3</td>
<td>1.000</td>
<td>3133</td>
<td>5.6</td>
<td>5.8</td>
<td>32.8</td>
</tr>
<tr>
<td>Configuration 4</td>
<td>0.248</td>
<td>3533</td>
<td>12.6</td>
<td>3.8</td>
<td>51.0</td>
</tr>
<tr>
<td>Configuration 5</td>
<td>0.500</td>
<td>3786</td>
<td>9.5</td>
<td>4.8</td>
<td>47.0</td>
</tr>
<tr>
<td>Configuration 6</td>
<td>0.333</td>
<td>3625</td>
<td>12.6</td>
<td>4.4</td>
<td>55.0</td>
</tr>
<tr>
<td>Configuration 7</td>
<td>0.166</td>
<td>3322</td>
<td>12.4</td>
<td>3.5</td>
<td>44.4</td>
</tr>
</tbody>
</table>

Table 4: Power and RPM measurements across different resistance load scenarios.

Note that the load encountered at the sprocket is 2.5X the load at the resistors due to the efficiency factor at the generator. Nevertheless, these observations were counter intuitive that the current drawn and hence the power consumed by the resistors increased only until the resistance decreased to 0.33Ω but then started to decrease even as the resistance decreased further. At this time, we do not have a sound explanation for this phenomenon and could be attributable to the current limitations of the power supply (max current drawn could be greater than the rating of the power supply and so current gets throttled) or the motor / generator current-output characteristics. However, interpreting this load behavior of these generators, power supply and motors is not in the scope of the study and so we chose to work with Configuration 1 – the configuration that resulted in the greatest load at the resistors and Configuration 3 – the least load on the resistors.

We determined the number of cycles to achieve failure as the average of two separate 3D printed sprockets each for Configurations 1 and 3 (four in total) and is reported in Table 5. Here are some of the highlights:
- Configuration 1 (55W load) failed on an average after 6 hours and 49 minutes which at 3637 RPM translates to 1,487,533 cycles (3637 RPM x 60 minutes x 6 hours and 49 minutes)
- Neither of the sprockets on Configuration 3 (32.8W load) failed even after 16 hours of operation and we refer to that as “Infinite Life” – a long duration of operation which is well sufficient for any real-life application. Note that with 5.5cm radius sprocket, at 3600 RPM and 16 hours of rotation and 1:1 gear ratio between the gear and wheel, it translates to a linear distance traversal of almost 120 kilometers, which for all practical purposes, and particularly for FRC robot related applications, is indeed “infinite life”. 16 hours also translates to about 3.5MM cycles
- In all these instances, the metal sprocket failed well before the 3D printed sprocket and we had to utilize multiple metal sprockets before the 3D printed sprocket failed.
- We interpreted the failure of the sprocket in the torsional failure mode as the failure at one cycle of operation. The average torsional failure load across two sprockets was 30.4 ft. lbs.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Load on the sprocket (W or ft. lbs.)</th>
<th>Load on the sprocket W(N)</th>
<th>Bending stress (MPa)</th>
<th>Number of cycles at failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Configuration 1</td>
<td>55W</td>
<td>6.565</td>
<td>2.4951</td>
<td>1.487MM</td>
</tr>
<tr>
<td>Configuration 3</td>
<td>32.8W</td>
<td>4.544</td>
<td>1.7274</td>
<td>Infinite life (did not experience failure after 3.5MM cycles)</td>
</tr>
<tr>
<td>At torsional failure</td>
<td>30.48 lbs.</td>
<td>22.032</td>
<td>8.375</td>
<td>1</td>
</tr>
</tbody>
</table>

Note that the bending stress of bulk PLA materials is around 50 MPa. Given these sprockets are at 10% infill, we can estimate the torsional failure bending stress of 100% infill materials to be 3X those of 10% infill sprockets (as supported by the torsional stress experiment results shown in Table 2) or around 25 Mpa. We believe a 50% reduction in the bending stress is reasonable if one were to compare bulk PLA to actual 3D printed PLA due to the magnitude of thermal stresses imposed on the latter. Given that, our results of observing 8.75 Mpa as the max bending stress is consistent with observations on bending stress in bulk materials. With these data points, a tradeoff curve can be surmised. With loads greater than for Configuration 1, additional data points can be plotted which will be items to explore in a future study.

3. Implications
The implications of this study are as follows:
- These 3D printed sprockets begin to demonstrate infinite life at loads somewhere between 6.5N and 4.5N of load.
- The results are also consistent with the results of the earlier experiments. The metal sprockets outperform 3D printed sprockets at torsional stress tests while 3D printed sprockets outperform the metal sprockets on fatigue loads.
- It is plausible that there is a load point where the metal and 3D printed sprockets have comparable performance in terms of number of load cycles before they failed, but that could be again be the topic of a future study.
G. Future Work

This study was meant to be just the beginning. We can envision numerous follow-on studies that can both build upon the work we have conducted in this study, and expand upon the study of the impact of 3D printed components.

1. Expand fatigue load tests

As discussed in the earlier section, the fatigue load tests can be extended to other load points perhaps with a different power supply and motors and compared against the curves for metal sprockets.

2. Varied Print Parameters

This study can also be extended to other 3D printer build parameters such as build plate temperature, nozzle temperature and cooling time (post printing) to analyze their effects on 3D printed part strength.

3. Other Materials

This study can also be expanded to other 3D printing materials such as ABS and to other technologies such as SLS to quantify the impact they have on performance.

4. Testing other types of failure modes and stresses:

Though we have tested the two critical failure modes we can extend into numerous other sources of failure and stresses such as impact, side shock, offset loads and so on.

5. Other Structures

A similar study can be carried out for other part shapes such as gears, axles, hinges, clips, etc.
H. Acknowledgements

I would like to acknowledge Mr. Andy Howell for his tireless guidance with the building of the manufacturing rig, as well as Ms. Rachel Gardner, CEO of Chap Research, and Mr. Eric Rothfus, Chief Mentor of Chap Research, for their valuable reviews and feedback. Last but not the least, I am deeply thankful to Drs. Carolyn Seepersad, Associate Professor of Mechanical Engineering at the University of Texas, Austin, Vikram Devarajan co-founder and CEO of Structured Polymers, Inc. and Jim Mikulak co-founder and VP of Engineering at Structured Polymers, for inspiring me to pursue research in the world of 3D printing.
I. References


2- http://www.firstinspires.org/


4- http://chapresearch.com/?page_id=189


Appendix A - Sprocket Production

The sprockets for these tests were generated by a program called SprocketR. SprocketR is a program created by Chase Probst at Chap Research. The program uses a sine function to generate sprockets. To use this program or learn more about it, visit www.ChapResearch.com.
Appendix B - Test Rig for Torsional Stress

Figure 12: SolidWorks model of torsional test rig.

A – Built from 2x4 wood block, 18 inches long; attached to B
B – Built from 2x4 wood block, 3 ft. long; attached to A, E and D
C – 1.125 inch hole that has an Andy-Mark hex bearing in it; a hex axle runs through the bearing with a sprocket mounted on the axle
D – Platform made from wood blocks to provide stability and support through the experiment
E – Basket utilized to bear the loads/weights during the experiment
F – Metal bar attached to the sprocket 1 ft. from the center of the sprocket; connects the sprocket to the basket that holds the weights
G – #25 Chain that needs to be kept in tension and fixed to the wood. Our construction included a simple spring mounted to D.

Materials List for Torsional Test Analysis
1. 6 feet of 2x4 wood
2. 2 X 2½" hex bore, flanged, heavy duty inner race shielded ball bearing (FR8ZZ-HexHD) (am-2986)
3. 3-4 feet of #25 chain
4. 6 inches of ½ inch hex
5. Versa Hub
6. 3 feet of 1x1 aluminum channel
7. Scrap metal bar
Appendix C – Test Rig for Fatigue Stress Analysis

Figure 13: SolidWorks model of fatigue test rig.

A – 18” Aluminum 1x1 channel
B – 60 tooth metal sprocket (never changes)
C – AndyMark Mini CIM (behind sprocket)
D – #25 chain that goes around both sprockets as shown
E – 3D printed sprocket that is being tested
F – Plate the sprocket is mounted to
G – Motor that is back-driven and placed on a load

Materials List for Fatigue Test Analysis
1. Aluminum 1x1 channel that is 18 inches long
2. 60 tooth metal sprocket, this sprocket never changes
3. AndyMark Mini CIM (Behind sprocket)
4. #25 chain that goes around both sprockets as shown
5. 3D printed sprocket that is being tested
6. Plate the sprocket is mounted to
7. Motor that is back-driven and placed on a load